Super MDS: Source Location from Distance and Angle Information

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***Abstract*— We consider the simultaneous localization of multiple sources from distance and angle information. An extension of the multidimensional scaling (MDS) technique is given, which allows for both distance and angle information to be processed algebraically (without iteration) and simultaneously. Simulations demonstrate the superiority the *super* MDS algorithm compared to conventional metric MDS, which relies only on Euclidean distances, and illustrate the impact that angle information may have on the accuracy of source localization. An advantage of the method is that localization under an absolute coordinate system is achievable with knowledge of the coordinates of a single node.**

I. Introduction

The introduction of location-based services is emerging as one of the major innovations of wireless and personal communications systems in the near future. Consequently, significant effort has been recently poured into the development of accurate localization algorithms.

Localization techniques can be classified in terms of deployability. Non-deployable techniques, such as fingerprinting and geo-regioning [1], [2], are those that require probing of the environment in which it is to operate. In contrast, deployable localization methods rely solely on ranging and angle measurements, acquired by the localization system itself, after deployment. In other words, deployable localization algorithms are techniques that map ranging and angle information onto the estimated coordinates of the sources to be localized.

Angle information can be obtained by estimating the angle of arrival (AoA) of signals at different receivers. On the other hand, ranging can be performed typically from time (difference) of arrival (ToA and TDoA, respectively) or received signal strength (RSS) estimates. Due to the fact that RSS-based ranging is highly subject to environmental conditions such as fading, shadowing and antenna distortion, time-based ranging methods are particularly attractive, especially since the advent of Ultra-Wideband Impulse Radio (UWB-IR) [3].

Since ranging and angle information are inevitably affected by errors, it is interesting to consider how the error processes corrupting these parameters impact on localization accuracy.

To the problem at hand, angle and ranging can be seen as complimentary (extrinsic) information, since the estimation of these quantities are affected by entirely different factors. With a low-frequecy narrowband signal, for instance, AoA estimation can be accurately performed and inexpensively even under non-line-of-sight (NLOS) conditions, provided that the multipath in the environment is not too dense [4].

In contrast, with a wideband signal, ToA-based high-precision ranging can be performed at low cost even in dense multipath environments, provided that line-of-sight (LOS) conditions exist [5].

It is reasonable to assume, therefore, that both functionalities will be simultaneously available in future wireless devices, so as to support higher accuracy localization functionalities. The challenge remains, nevertheless, in developing localization algorithms that can efficiently exploit angle and ranging information. In this paper one such an algorithm is proposed, where angle and ranging information are processed jointly in a natural and straightforward manner.

The algorithm is based on the multidimensional scaling (MDS) technique, an algebraic method to classify abstract entities according to a measure of mutual dissimilarities which was brought to the context of multi-source localization in [6].

The work in [6] is based on the classic (metric) MDS [7], where the entities are the source coordinates and the dissimilarity measure the Euclidean distance between a pair of sources. Our contribution is a new formulation of the MDS-based localization problem, where the entities are the *vectors* pointing from a source to another, and the measure of dissimilarity their inner product.

In a Graph-Theoretical setting, the technique proposed in [6] translates to modeling the network as an undirected graph $G_{\eta,N}(\mathrm{X},\mathrm{V},\mathrm{D})$, embedded in a η-dimensional space, where the entities are its N vertices V, and the dissimilarities its weights D. Conversely, in our formulation, the network is modeled as a complete oriented graph (also know as a *tournament*), where the entities are the *edges* V of the graph, and the dissimilarity metric their inner product[1].

The new MDS formulation is referred to as *super* MDS, and offers the following advantages over the technique proposed in [6]. First, the new formulation permits a unified processing of angle and ranging information. Second, it leads to a simplified structure of the positive semi-definite Gram Kernel matrix at the hart of the MDS algorithm [8].

In allusion to its construction, the Gram Kernel used in our Super-MDS algorithm is referred to as the Edge Gram Kernel. A fortunate property of the Edge Gram Kernel is that it is each of its elements depends only on one angle and a single pair of distance estimates.

[1]In our context edge orientations are arbitrary and necessary only so as to allow for the definition of an edge inner product in similar terms to that of elementary vector algebra.

In contrast, the Gram Kernel used in the metric MDS is computed by applying a double-centering transformation [8] onto the Euclidean distance matrix (EDM), such that each of its elements depends on *all* the distance measurements. In short, the Edge Gram Kernel is, by construction, far more robust to the propagation of measurement errors, which may have devastating effects on the performance of the MDS-based localization algorithm subject to noisy and biased distance estimates [9].

The remainder of the paper is organized as follows. First, in section II metric MDS formulation of the multi-source localization problem is reviewed. The Edge Gram Kernel and the subsequent novel formulation of the MDS-based localization algorithm is discussed in section III. In section IV simulation examples and performance comparisons are given, followed by conclusions in section V.

II. Source Localization and Metric MDS

Consider a network of N devices in an η-dimensional Euclidean space. Such a network can be represented by a undirected graph $G_{\eta,N}(\mathrm{X},\mathrm{V},\mathrm{D})$, with vertices $\mathrm{X}=\{\mathbf{x}_1,\ldots,\mathbf{x}_N\}$, undirected edges $\mathrm{V}=\{\mathbf{v}_{n,m}\}$, and weights $\mathrm{D}=\{d_{n,m}\}$, where $n,m\in\{1,\cdots,N\}$.

Here, edges represent communication links, while weights are the Euclidean distance between sources, given by

$$d_{n,m}=\mathcal{D}(\mathbf{x}_n,\mathbf{x}_m)\triangleq\sqrt{\langle(\mathbf{x}_n-\mathbf{x}_m);(\mathbf{x}_n-\mathbf{x}_m)\rangle},\qquad(1)$$

where $\langle\mathbf{x}_n;\mathbf{x}_m\rangle$ denotes the inner product of $\mathbf{x}_n$ and $\mathbf{x}_m$.

To the undirected graph G is associated the EDM $\mathbf{D}\triangleq[d_{n,m}]\in\mathbb{S}^{N\times N}$, where $\mathbb{S}^{N\times N}$ denotes the set of N-by-N hollow symmetric matrices with positive real entries.

Hereafter, a matrix $\mathbf{S}\in\mathbb{S}^{N\times N}$ will be referred to as a *true* EDM if and only if (iff)

$$\exists\,\mathbf{X}=[\mathbf{x}_1,\cdots,\mathbf{x}_N]\in\mathbb{R}^{\eta\times N}\mid\mathbf{S}=\mathcal{D}(\mathbf{X}).\qquad(2)$$

A matrix $\tilde{\mathbf{D}}\in\mathbb{S}^{N\times N}$ that is not a true EDM, but is close to the EDM $\mathbf{D}$ in the Frobenius norm sense is referred to as a *sample* of $\mathbf{D}$, or simply, as an EDM sample. In our context, since EDM sample matrices are constructed from ranging measurements, the symmetry property of $\tilde{\mathbf{D}}$ can always be achieved by averaging the pair of distance estimates $d_{n,m}$ and $d_{m,n}$, obtained at the vertices $\mathbf{x}_n$ and $\mathbf{x}_m$, respectively.

Consider the following transformation of the EDM $\mathbf{D}$

$$\mathbf{K}_{\mathbf{D}}\triangleq\mathcal{K}(\mathbf{D})=-\frac{1}{2}\mathbf{J}\cdot\mathbf{D}^{\odot2}\cdot\mathbf{J},\qquad(3a)$$

$$\mathbf{J}\triangleq\mathbf{I}-\frac{1}{N}(\mathbf{1}\cdot\mathbf{1}^{\mathrm{T}}),\qquad(3b)$$

where $^{\odot m}$ denotes the m-th element-wise (Hadamard) power and $\mathbf{1}$ is a vector whose entries are all one.

It can be shown [8] that the positive semi-definite matrix double-centered kernel $\mathbf{K}_{\mathbf{D}}$ is a rotation of the Gram matrix

$$\mathbf{K}_{\mathbf{X}}\triangleq\mathbf{X}^{\mathrm{T}}\cdot\mathbf{X},\qquad(4)$$

where $^{\mathrm{T}}$ denotes transpose.

Consider the matrix $\hat{\mathbf{X}}$, retrieved from $\mathbf{D}$ through,

$$\hat{\mathbf{X}}=\left(\mathbf{U}_{\mathbf{D}}\Big\rfloor_{N\times\eta}\cdot\mathbf{\Lambda}_{\mathbf{D}}\Big\rfloor_{\eta\times\eta}^{\odot\frac{1}{2}}\right)^{\mathrm{T}},\qquad(5)$$

where $\cdot\rfloor_{n\times m}$ denotes n-by-m upper-left partition and the matrices $\mathbf{U}_{\mathbf{D}}$ and $\mathbf{\Lambda}_{\mathbf{D}}$ are, respectively, the eigenvector and eigenvalue matrices of $\mathbf{K}_{\mathbf{D}}$, $i.e.$,

$$\mathbf{K}_{\mathbf{D}}=\mathbf{U}_{\mathbf{D}}\cdot\mathbf{\Lambda}_{\mathbf{D}}\cdot\mathbf{U}_{\mathbf{D}}^{\mathrm{T}},\qquad(6)$$

The equivalence of $\mathbf{K}_{\mathbf{D}}$ and $\mathbf{K}_{\mathbf{X}}$ implies that the matrix $\hat{\mathbf{X}}$ is related to the coordinate matrix $\mathbf{X}$, by a rotation, translation and reflections.

Therefore, if at least $A=\eta+1$ columns of $\mathbf{X}$ are known – which without loss of generality can be assumed to be the first ones – the entire matrix $\mathbf{X}$ can be reconstructed from $\hat{\mathbf{X}}$ through the Procrustes transformation [10],

$$\mathbf{X}=\alpha\cdot\hat{\mathbf{X}}\cdot\mathbf{B}+\mathbf{c}\otimes\mathbf{1}_{[N\times1]},\qquad(7)$$

where $\otimes$ denotes Kronecker product and,

$$\alpha=\mathrm{tr}\Big([\mathbf{K}_A^{\mathrm{T}}\cdot\mathbf{K}_A]^{\frac{1}{2}}\Big)\cdot\frac{\|\mathbf{Z}_A\|_{\mathrm{F}}}{\|\hat{\mathbf{Z}}_A\|_{\mathrm{F}}},\qquad(8)$$

$$\mathbf{B}=\mathbf{U}_A^{\mathrm{T}}\cdot\mathbf{U}_A,\qquad(9)$$

$$\mathbf{c}=\frac{1}{A}\cdot\sum_{a=1}^{A}\mathbf{x}_a-\frac{\alpha}{A}\cdot\left[\sum_{a=1}^{A}\hat{\mathbf{x}}_a\right]\cdot\mathbf{B},\qquad(10)$$

where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm and,

$$\mathbf{K}_A=\mathbf{Z}_A^{\mathrm{T}}\cdot\hat{\mathbf{Z}}_A=\mathbf{U}_A\cdot\mathbf{\Lambda}_A\cdot\mathbf{U}_A^{\mathrm{T}},\qquad(11)$$

$$\mathbf{Z}_A=\mathbf{X}_A-\frac{1}{A}\cdot\left[\sum_{a=1}^{A}\mathbf{x}_a\right]\otimes\mathbf{e}_{[A\times1]},\qquad(12)$$

$$\hat{\mathbf{Z}}_A=\hat{\mathbf{X}}_A-\frac{1}{A}\cdot\left[\sum_{a=1}^{A}\hat{\mathbf{x}}_a\right]\otimes\mathbf{e}_{[A\times1]}.\qquad(13)$$

The algebraic procedure described above is exact, provided that $\mathbf{D}$ is a true EDM matrix. In practice, however, one can only obtain an EDM sample $\tilde{\mathbf{D}}$, corrupted by noise and bias[2], which obviously introduces errors in solution.

This is the first weakness of the metric MDS-based localization algorithm. In particular, the Gram Kernel computed using equation (3a) is so structured that each of its elements depends on all the entries of $\tilde{\mathbf{D}}$, resulting in an error-propagation-prone algorithm.

A second weakness of the technique is the inability to process angle information. In the following section, a novel formulation of the MDS-based localization problem is given, which not only allows for ranging and angle information to be processed jointly, but also is less prone to error propagation.

[2]Although not accounted for here, erasures may also occur, but can be dealt with by employing a completion technique such done in [9], [11] and references thereby.

III. Source Localization and Super MDS

Let us associate our network to a complete oriented graph $G_{\eta,N}(\mathrm{X},\vec{\mathrm{V}},\mathrm{D})$, where $\vec{\mathrm{V}}=\{\vec{\boldsymbol{v}}_{n,m}\}$ is the set of edges with arbitrary, but unique, orientations (vectors). For convenience, the orientation of all edges is chosen so that we may order the $M=N(N-1)/2$ elements of $\vec{\mathrm{V}}$ progressively, *i.e.*, $\vec{\mathrm{V}}=\{\vec{\boldsymbol{v}}_{1,2},\vec{\boldsymbol{v}}_{1,3},\cdots,\vec{\boldsymbol{v}}_{N-1,N}\}$.

In order to simplify the notation, let us also relabel each of the mutually exclusive pairs of indexes (n,m), with $n<m$, by a different number i, such that the edge departing from the vertex $\mathbf{x}_n$ to the vertex $\mathbf{x}_m$ can be denoted simply by,

$$\vec{\boldsymbol{v}}_i=(\mathbf{x}_m-\mathbf{x}_n)=[(x_{m,1}-x_{n,1}),\cdots,(x_{m,\eta}-x_{n,\eta})]^{\mathrm{T}}. \quad (14)$$

Next, define the dissimilarity metric $k_{i,j}$ of the i-th and j-th edges as the inner product,

$$\begin{aligned} k_{i,j} &\triangleq \langle\vec{\boldsymbol{v}}_i;\vec{\boldsymbol{v}}_j\rangle=\langle(\mathbf{x}_m-\mathbf{x}_n);(\mathbf{x}_q-\mathbf{x}_p)\rangle \\ &= d_{n,m}d_{p,q}\cos(\theta_{i,j}). \end{aligned} \quad (15)$$

The set of all dissimilarity measures $k_{i,j}$, corresponding to all pairs of edges in the graph, can be conveniently assembled into the kernel matrix

$$\begin{aligned} \mathbf{K} &= \langle[\vec{\boldsymbol{v}}_1,\cdots,\vec{\boldsymbol{v}}_M]^{\mathrm{T}};[\vec{\boldsymbol{v}}_1,\cdots,\vec{\boldsymbol{v}}_M]\rangle \\ &= \mathbf{V}\cdot\mathbf{V}^{\mathrm{T}}=\begin{bmatrix} \langle\vec{\boldsymbol{v}}_1;\vec{\boldsymbol{v}}_1\rangle & \cdots & \langle\vec{\boldsymbol{v}}_1;\vec{\boldsymbol{v}}_M\rangle \\ \vdots & \ddots & \vdots \\ \langle\vec{\boldsymbol{v}}_M;\vec{\boldsymbol{v}}_1\rangle & \cdots & \langle\vec{\boldsymbol{v}}_M;\vec{\boldsymbol{v}}_M\rangle \end{bmatrix}. \end{aligned} \quad (16)$$

The matrix $\mathbf{V}$ that appears in equation (16) will be referred to as the *vector* matrix. Notice that $\mathbf{V}$ has the structure of a coordinate matrix associated to the M points that define the vectors $\boldsymbol{v}_i$, $i=\{1,\cdots,M\}$. It follows that $\mathbf{K}$ is, by construction, a Gram matrix of rank η and, therefore, this matrix will be hereafter referred to as the Edge Gram Kernel.

It is also if interest to notice that since $k_{i,j}$ depends only on a pair of distances and the angle between the corresponding vectors, samples of the Edge Gram Kernel can be constructed directly from measured distances and angles, eliminating the requirement of a double-centering procedure – such as that of equation (3a) – and, therefore, limiting the propagation of errors affecting a given observation to the entire kernel.

The Super MDS algorithm results directly from the fact that $\mathbf{K}$ is a Gram matrix. Indeed, consider

$$\hat{\mathbf{V}}=\Big(\underline{\mathbf{U}}\rfloor_{M\times\eta}\cdot\underline{\mathbf{\Lambda}}\rfloor_{\eta\times\eta}^{\odot\frac{1}{2}}\Big)^{\mathrm{T}}, \quad (17)$$

$$\mathbf{K}=\mathbf{U}\cdot\mathbf{\Lambda}\cdot\mathbf{U}^{\mathrm{T}}. \quad (18)$$

The matrix $\mathbf{V}$ can be retrieved from $\hat{\mathbf{V}}$ by using the Procrustes transformation, in similarity to what is described by equations (7) through (13).

Notice, however, that in this case the Procrustes transformation requires that $\eta+1$ rows of $\mathbf{V}$ – as opposed to the coordinates of $\eta+1$ sources – are known. This condition is relaxed compared to that of the metric MDS-based localization algorithm [6], since each vector $\boldsymbol{v}_{n,m}$ can – unlike the coordinates of $\mathbf{x}_n$ and $\mathbf{x}_m$ – be estimated by measuring the distance $d_{n,m}$ and the AoA of the m-th source at n.

The practical implication of this property of the Super MDS-based algorithm is, as it will soon be shown, that *a priori* knowledge of the coordinates of a single source is sufficient to achieve multi-source localization with absolute (global) orientation.

Indeed, the final step of the localization algorithm is to recover an estimate the coordinate matrix $\mathbf{X}$ from the retrieved vector matrix $\mathbf{V}$.

To this end, consider the following system of linear equations, derived directly from equation (14),

$$\mathbf{C}\cdot\mathbf{X}=\mathbf{V}, \quad (19)$$

where the coefficient matrix $\mathbf{C}$ has the block upper-triangular structure shown below

$$\mathbf{C}=\left[\begin{array}{c:c:c} \mathbf{1}_{N-1\times 1} & \multicolumn{2}{c}{-\mathbf{I}_{N-1\times N-1}} \\ \hdashline \mathbf{0}_{N-2\times 1} & \mathbf{1}_{N-2\times 1} & -\mathbf{I}_{N-1\times N-1} \\ \hdashline & \ddots & \ddots \quad \ddots \\ \hdashline \multicolumn{2}{c:}{\mathbf{0}_{1\times N-2}} & 1 \;\vdots\; -1 \end{array}\right]. \quad (20)$$

It is trivial to prove[3] that $\mathrm{rank}(\mathbf{C})=N-1$, such that the system given in equation (19) is under-determined by one equation. In fact, any partition of $N-1$ linearly independent rows of $\mathbf{C}$ has rank $N-1$.

In other words, if it is assumed that the absolute coordinates of a single source (which without loss of generality can be taken to be the first) is known, a solution of the Super-MDS problem can be found by considering any of the following reduced systems

$$\left[\begin{array}{c:c} 1 & \mathbf{0}_{1\times N-1} \\ \hdashline \multicolumn{2}{c}{[\mathbf{C}]_{N-1\times N}} \end{array}\right]\cdot\left[\begin{array}{c} \mathbf{x}_1 \\ \hdashline [\mathbf{X}]_{N-1\times\eta} \end{array}\right]=\left[\begin{array}{c} \mathbf{x}_1 \\ \hdashline [\mathbf{V}]_{N-1\times\eta} \end{array}\right], \quad (21)$$

where $[\mathbf{C}]_{N-1\times N}$ denotes an $N-1$ row-partition of $\mathbf{C}$ with rank $N-1$, while $[\mathbf{X}]_{N-1\times\eta}$ and $[\mathbf{V}]_{N-1\times\eta}$ denote the corresponding $N-1$ row-partitions of $\mathbf{X}$ and $\mathbf{V}$, respectively.

Consider the following expression for the inverse of block matrices [12],

$$\left[\begin{array}{c:c} x & \mathbf{y} \\ \hdashline \mathbf{z} & \mathbf{W} \end{array}\right]^{-1}=\left[\begin{array}{c:c} (x-\mathbf{y}\cdot\mathbf{W}^{-1}\cdot\mathbf{z}) & -x\cdot\mathbf{y}\cdot(\mathbf{W}-x\cdot\mathbf{z}\cdot\mathbf{y})^{-1} \\ \hdashline -\mathbf{W}^{-1}\cdot\mathbf{z}\cdot(x-\mathbf{y}\cdot\mathbf{W}^{-1}\cdot\mathbf{z}) & (\mathbf{W}-x\cdot\mathbf{z}\cdot\mathbf{y})^{-1} \end{array}\right], \quad (22)$$

where x is a scalar, $\mathbf{y}$ and $\mathbf{z}$ row- and column-vectors, and $\mathbf{W}$ an invertible matrix.

It is clear from equation (22) that any of the linear systems described by equation (21) is determined, because $\mathbf{W}$ is an $N-1$ column-partition of $[\mathbf{C}]_{N-1\times N}$, which has rank $N-1$.

A particularly interesting case is the one where $[\mathbf{C}]_{N-1\times N}$ is given by the first $N-1$ rows of $\mathbf{C}$, which leads to

$$\left[\begin{array}{c:c} 1 & \mathbf{0}_{1\times N-1} \\ \hdashline \mathbf{1}_{N-1\times 1} & -\mathbf{I}_{N-1\times N-1} \end{array}\right]\cdot\mathbf{X}_{N\times\eta}=\left[\begin{array}{c} \mathbf{x}_1 \\ \hdashline \mathbf{V}_{N-1\times\eta} \end{array}\right]. \quad (23)$$

[3]Proof: Let $\mathbf{C}_i$ denote the i-th $N-i$ row-partition of $\mathbf{C}$, such that we may write $\mathbf{C}=[\ \mathbf{C}_1^{\mathrm{T}} \;\vdots\; \cdots \;\vdots\; \mathbf{C}_{N-1}^{\mathrm{T}}\ ]^{\mathrm{T}}$. Then, simply notice that for all $1\leqslant i<N$, the partition $\mathbf{C}_{i+1}$ is given by the last $N-i-1$ rows of $\mathbf{C}_i$, subtracted by its first row.

From equation (22) it is found that the coefficient matrix of this reduced linear system is invariable to the inverse operation, so that the solution of the Super-MDS becomes simply

$$\mathbf{X}_{N\times\eta}=\left[\begin{array}{c:c}1 & \mathbf{0}_{1\times N-1}\\ \hdashline \mathbf{1}_{N-1\times 1} & -\mathbf{I}_{N-1\times N-1}\end{array}\right]\cdot\left[\begin{array}{c}\mathbf{x}_1\\ \hdashline \mathbf{V}_{N-1\times\eta}\end{array}\right]. \quad (24)$$

The fact that only N–1 rows of $\mathbf{C}$ suffice to solve the super MDS localization problem is obviously a direct consequence of the fact that there are only N–1 linearly independent vectors $\boldsymbol{v}_i$ in $\mathbf{V}$ which, in turn, means that the super MDS algorithm can be formulated over an $N-1$-by-$N-1$ reduced Edge Gram Kernel.

$$\bar{\mathbf{K}}=[\mathbf{V}]_{N-1\times\eta}\cdot[\mathbf{V}]^{\mathrm{T}}_{N-1\times\eta}, \quad (25)$$

where $[\mathbf{V}]_{N-1\times\eta}$ denotes the $N-1$ row-partition of $\mathbf{V}$, containing only linearly independent vectors.

The Super-MDS moulti-source localization algorithm can be summarized as follows:

Inputs:

1. *Pairwise distance and angle estimates.*
2. *The coordinates of one referential source.*

Steps:

1. *Construct the reduced Edge Gram Kernel* $\bar{\mathbf{K}}$ *using equation* (25).
2. *Compute the eigen-decomposition of* $\bar{\mathbf{K}}$.
3. *Estimate the edge vector* $[\hat{\mathbf{V}}]_{N-1\times\eta}$ *using equation* (17).
4. *Retrieve* $[\mathbf{V}]_{N-1\times\eta}$ *from* $[\hat{\mathbf{V}}]_{N-1\times\eta}$ *via the Procrustes transformation* (7).
5. *Compute the coordinate estimates by solving the linear system described in equation* (21)

IV. SIMULATIONS

In this section, the performance of the super MDS technique described above is compared against the classic metric MDS algorithm, which utilizes only distance information.

The simulations are designed to provide insight on how much accuracy can be gained by utilizing angle information or, conversely, how much accuracy is needed from angle estimates in order to reap better accuracies in comparison to using only distance measurements.

In each realization, a fully connected random network of sensors Gaussianly distributed in the 3-dimensional Euclidean space, with statistical center at the origin and a unitary spread (variance) is generated. All metric units are normalized and, therefore, shall be omitted henceforth.

Range estimations $\tilde{d}$, *i.e.*, the noisy measurements of the distance between each pair of sensors, are modeled as Gamma-distributed random variables with the mean given by the true distance d and a standard deviation σ_d related to the ranging error affecting its measurement [13], *i.e.*,

$$p_\Gamma(\tilde{d};\alpha,\beta)=\frac{1}{\beta^\alpha\Gamma(\alpha)}\cdot\tilde{d}^{(\alpha-1)}\cdot e^{\frac{\tilde{d}}{\beta}}, \quad (26)$$

where

$$\alpha=d^2/\sigma_d^2, \quad (27)$$

$$\beta=\sigma_d^2/d. \quad (28)$$

The complete EDM sample corresponding to these Gamma-distributed distance estimates are then fed to the metric MDS localization algorithm described in II.

In turn, angle estimation errors are modeled as random processes distributed according to the Tikhonov probability density function (pdf) [14], [15]

$$p_{\mathrm{T}}(\theta;\rho)=\frac{1}{2\pi I_0(\alpha)}\cdot e^{\rho\cos(\theta)},\ \theta\in[-\pi,\pi],\ \rho\geqslant 0. \quad (29)$$

The parameter ρ controls the shape of this pdf, such that $p_{\mathrm{T}}(\theta;\rho)$ tends to a uniform distribution for $\rho\to 0$, and to a Dirac delta at 0 when $\rho\to\infty$. For a given ρ, we shall define the angle error ε_θ as the limiting angle that encloses 90% of the area below $p_{\mathrm{T}}(\theta;\rho)$, *i.e.*,

$$\varepsilon_\theta=\theta_{\mathrm{L}}\ \Bigg|\ \int_{-\theta_{\mathrm{L}}}^{\theta_{\mathrm{L}}}p_{\mathrm{T}}(\theta;\rho)\,d\theta=0.9. \quad (30)$$

The range and angle estimates perturbed according to the Gamma and Tikhonov distributions are then fed to the super MDS localization algorithm described in section III.

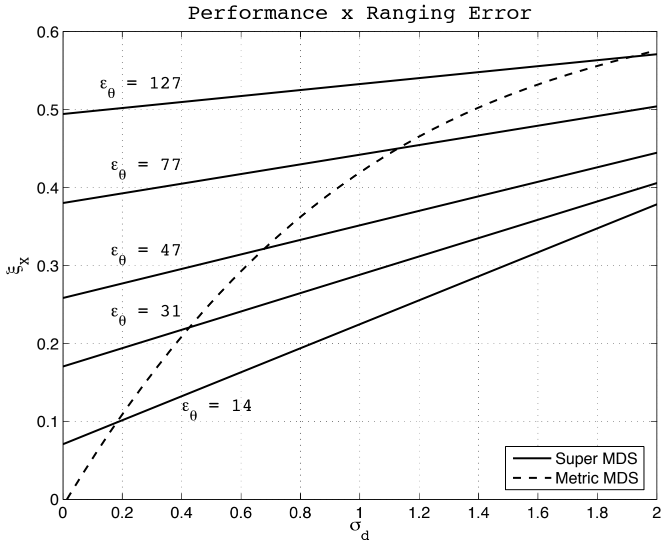
The performance of the metric and super MDS localization algorithms can be measured by the Frobenius norm of the difference between the estimate coordinate vector $\hat{\mathbf{X}}$ and the corresponding true coordinate vector $\mathbf{X}$, *i.e.*,

$$\xi_{\mathbf{X}}=\|\hat{\mathbf{X}}-\mathbf{X}\|_{\mathrm{F}}. \quad (31)$$

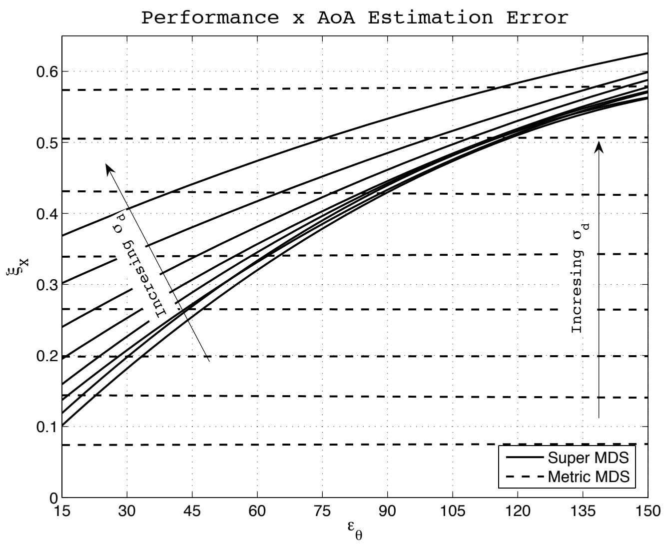
The classic (metric) MDS and super MDS algorithms are compared in figure 1. First, the estimation accuracies attained by both algorithms as functions of the ranging error are shown in figure 1(a). In the case of the super MDS algorithm, the curves are parameterized by the angle error ε_θ.

The plots show that the localization accuracy of the super MDS algorithm grows (linearly) with σ_d at a lower pace than that of the metric MDS in the interval of interest (of moderate ranging errors). In other words, it is found angle information, even when not highly accurate, can significantly improve the accuracy of MDS-based localization algorithms in the presence of severe ranging errors. For example, in a scenario with $\sigma_d=2$ and $\varepsilon_\theta\approx 31°$, the super MDS algorithm exhibits the same performed attained by the metric MDS when the range estimation errors are inferior to 1.

The fact that angle information, even if imperfect, can be utilized to increase the robustness of to ranging errors can be further appreciated in figure 1(b), where the localization accuracy of the super MDS algorithm as a function of AoA estimation errors and parameterized by σ_d – varying from 0.1 to 10 – are compared against the accuracy of the corresponding metric MDS. As the plots illustrate, while the accuracy of both algorithms are affected by an increase on the range estimation error, the relative performance degradation suffered by the super MDS algorithm (for any given underlying ε_θ) is significantly lower than that experienced by its metric counterpart.



(a) $\xi_{\mathbf{X}}$ as a function of σ_d.



(b) $\xi_{\mathbf{X}}$ as a function of ε_θ.

Fig. 1. Performance of metric and super MDS as a function of σ_d and ε_θ.

V. Conclusions

In this paper, the simultaneous localization of multiple sources from imperfect ranging and AoA information was considered. A new formulation of the multidimensional scaling (MDS) technique that allows for angle information to be processed in combination to distance estimates was provided. The resulting algorithm, which is referred to as *super*-MDS, is shown to out-perform the classic (metric) MDS algorithm, especially in the presence of moderate ranging estimation errors, such as those characteristic of ranging technologies based on time of arrival (ToA) estimation with ultra-wideband technology [16].

All this is achieved at a slightly lower complexity than that of the metric MDS, in which the super MDS technique eliminates the need of computing a "double-centered" kernel matrix, and can be implemented using a reduced Gram kernel of size $N-1$-by-$N-1$.

A by-product of this feature is that error propagation is largely avoided, since the Edge Gram Kernel is computed (element-by-element), straight from the distance and angle estimates. Finally, another advantage of the method is that localization under an absolute coordinate system is achievable with knowledge of the coordinates of a single node.

The main disadvantage of super MDS is the requirement that nodes perform AoA estimation. This condition can, nevertheless, be significantly relaxed by the introduction of a matrix completion algorithm that can fill-in angle and distance information not measured by the network. The authors have already developed such a technique which is applicable to reconstruct missing distance information (see [9], [11]), and are currently pursuing an extension to the case of interest here.

It is worth mention that since the super and metric MDS algorithms have comparable computational complexities, it is foreseeable that super MDS technique can be utilized to track multiple targets of unequal and non-stationary vis-à-vis [17].

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