Mutual Information of Amplify-and-Forward DSTBCs over the Random Set Relay Channel

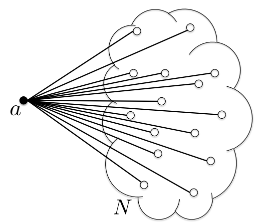
Qiang Xue and Giuseppe Abreu
Centre for Wireless Communications
University of Oulu, Oulu, Finland
P.O.Box 4500 FIN-90014
Email: [xueqiang,giuseppe]@ee.oulu.fi

Behnaam Aazhang
Dept. of Electrical & Computer Engineering
Rice University, Houston, USA
TX 77251-1892
Email: aaz@rice.edu

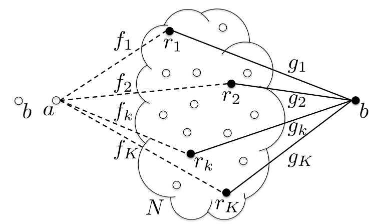
***Abstract*—We study the performance of amplify-and-forward (AF) distributed space-time block coded (DSTBC) cooperative systems over the random set relay channel (RSRC). Information transmission consists of two stages: first source to relays and then relays to destination. All terminals operate in half-duplex mode. Each of N available relays is allowed to decide independently whether to relay the source's information or not, based on its own instantaneous received signal-to-noise ratio (SNR). We derive mutual information (MI) for linearly dispersed full-rate-full-diversity DSTBCs over the random set relay channel with Rayleigh fading per branch and constant average transmit power *per* relay. Perfect interleaving is assumed to make the channel ergodic. The system concept is interesting in which no coordination amongst relays is required, and only backward channel state information (CSI) is assumed. Firstly we study MI of AF-DSTBCs over the RSRC with equal power allocation per node. The result reveals that a substantial gain in MI is achieved by the autonomic relaying mechanism, compared to the alternative of full-time-all-relay cooperation, despite the fact that the total receive power at the receiver increases with the number K of active relays. Maximum MI in this case fits very well as a simple-formed function of N. Then we fix the total average transmit power and investigate the effect of power balance between the two transmission stages. It is shown that for full-time all-relay cooperation the optimal strategy is to allocate power equally between the two stages [1]. However, for backward-CSI-based relay selection scheme, the optimal power balance is around $2{:}3$. Finally, we compare constellation-constrained MI and the unconstrained MI.**

I. Introduction

Studies about wireless relay networks with topology illustrated in figure 1 are conducted from several aspects. In [2], perfect CSI is assumed at relays, the transmitter and the receiver; each node has its own power constraint. In this scenario, the best scheme is to do network beamforming. The optimal power control strategy is derived to maximize the receive SNR for each channel realization. In [3], an opportunistic relaying scheme is proposed: the relay in the best channel state(in the sense of combining both backward and forward channel state) is selected with the aid of a protocol; aggregate power constraint is assumed. It is shown that opportunistic AF relaying is outage-optimal among single-relay selection methods. When only backward CSI is assumed at relays and the receiver, DSTBCs could be implemented to exploit the spacial diversity of such relay networks [4], [1], [5], [6], etc. In most studies about DSTBCs, it is assumed that all N relays will cooperate all the time. However, in AF relaying strategy, a relay does not only transmit amplified version of received signals but also unavoidably transmit amplified noise. So a relay in deep fading channel could transmit weak signals with strong noise, which degrades overall performance. Invoking this hand-waving reasoning, we propose a relay selection scheme based only on backward CSI. This scheme requires no extra CSI and is proved to outperform full-time-all-relay DSTBC scheme [7]. Exact bit error rate (BER) of M-ary PSK and QAM is derived for backward-CSI-based relay selection scheme and is used as performance metric in [7]. However, further analysis is hard to conduct with BER due to the complexity of the formulae.



(a) Stage 1.



(b) Stage 2.

Fig. 1. Illustration of the system concept. Solid (black) dots represent active nodes and empty (white) inactive ones. Due to the autonomy of relay selection, the number K of active relays appears to be random to he receiver.

In this paper, MI is derived and used as the tool of analysis. As will be seen from the SNR expression later, for each channel realization the channel is equivalent to an additive white gaussian noise (AWGN) channel whose capacity is well known. In [8], it was shown that for a fading channel if the channel sequence is i.i.d. and the input distribution which maximizes mutual information is the same regardless of the channel state, the capacity of the channel with CSI only at the receiver is given by the expectation of capacity of each channel realization. The constraint is satisfied in this scenario, so to obtain MI we first derive capacity for each channel realization, and then derive its expectation with respect to fading coefficients. The exact expression of average MI is derived for 1 active relay case. For cases with more than 1 active relays, a simple and asymptotically close upper bound is implemented. With the obtained formulae, MI as function of N and ξ which is the threshold on backward instantaneous SNR is investigated separately under the assumption of equal power constraint per terminal. Then, we fix the total average transmit power and compare different power balance between the two transmission stages. It is found that with all-time-full-relay scheme the best strategy is to assign power equally between the two stages, which accords with the conclusion in [1]; with optimal ξ, the optimal power balance is around 2:3 in this channel setting. At the end, constrained MI over PSK and QAM signals derived from BER formulae in [7] is compared with the unconstrained MI. It is found that an ideal DSTBC (with optimized modulation) comes to roughly 2 bits loss per dimension pair from the unconstrained MI.

The remainder of this paper is organized as follows. Section II describes the system model. MI is derived in section III. Results and discussion are given in section IV. The final section is our main conclusion.

II. System Model

Consider a wireless network as illustrated in figure 1 where a source node denoted by a communicates with a destination node denoted by b through the help of a pool of N relay nodes, with no direct link between a and b. In particular, we focus on single-antenna nodes and study a 2-stage amplify-and-forward autonomic and opportunistic relaying mechanism described as follows. In the first stage, the source broadcasts a symbol vector $\boldsymbol{s}$ to all N relays, in similarity to the systems proposed in [1], [5], [6], [9]. In the second stage however, we take inspiration from [3], [4], [10]–[12] and consider the case where only a subset of $K(0 \leq K \leq N)$ relays – hereafter referred to as *active* relays – amplify and forward the source's signals to the destination, employing a hypothetical linear DSTBC with rate ρ and diversity order η. For conciseness, we shall denote the set of active relays by $\mathcal{R} = \{r_1, \cdots, r_K\}$.

Let f_k and g_k denote the instantaneous complex-valued coefficients of the block-fading channels from a to the k-th active relay r_k, and from r_k to b, respectively, which are assumed to be Rayleigh-distributed with $\mathrm{E}[|f_k|^2]=\mathrm{E}[|g_k|^2]=1$. Assume also that the noise at all r_k's and b are independent and identically distributed (i.i.d.) zero-mean complex-valued Gaussian variates with variances σ_1^2 for all k, and σ_2^2 for b. Without loss of generality, assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

Let P_1 and γ_{ar_k} denote the average transmit power of the source and the SNR at the k-th relay, respectively. Under these assumptions, γ_{ar_k} is identical for all k, so that we may define $\gamma_{ar} \triangleq \gamma_{ar_k} = P_1/\sigma_1^2$. Finally, consider that a relay is activated if and only if (iff) its instantaneous SNR exceeds a given threshold ζ or, equivalently, iff $|f_k|^2 \geq \xi$, where $\xi = \zeta/\gamma_{ar}$. The resulting relay channel from the source a to the destination b, described by $\mathcal{C} \triangleq \{f_1 g_1, \cdots, f_K g_K\}$, is mathematically known as a *random set*, *i.e.* a set of random variates with random cardinality. Hereafter, we shall refer to $\mathcal{C}$ simply as the *random set relay channel*.

Under the assumption that the ensemble $\mathcal{F} = \{f_1, \cdots, f_K\}$ is uncorrelated, the probabilities $\Pr\{|f_k|^2 \geq \xi\}$ are independent for all k and therefore, the probability of having K active relays at a given channel realization is given by the binomial distribution

$$p_K(\xi, N) = \binom{N}{K} e^{-K\xi}(1 - e^{-\xi})^{(N-K)}, \qquad (1)$$

where $\binom{N}{K}$ is the binomial coefficient.

Hereafter, the average number of active relays that result with a given threshold will be denote by the symbol κ.

Although the described AF-DSTBC system requires instantaneous backward channel state information (IB-CSI) at relays for the purpose of autonomic relay selection, it was shown in [5], [6] that in order to achieve full diversity it is sufficient to use SB-CSI for amplification. Therefore, we consider the case where each active relay amplifies the source's signals with a constant scaling factor

$$\rho = \sqrt{\frac{P_2}{(1+\xi)P_1 + \sigma_1^2}}, \qquad (2)$$

where P_2 is the average transmit power of *each* relay. Notice that this implies that the total power transmitted from $\mathcal{R}$ to b increases linearly with K. It will be later shown that, despite the decreased receive power resulting from the autonomic relay selection mechanism of our system model, the MI achieved at the receiver is larger than that achieved under all-relay cooperation.

Next we abstract from code and decoder design challenges[1] and assume that the K active relays are capable of collectively transmitting the source's symbol vector s utilizing a hypothetical (ideal) full-rate DSTBC that provides full diversity gain. Then after each block transmission, given a signal s is transmitted the received signal y at destination is:

$$y = \rho\sqrt{P_1 \sum_{k=1}^{K} |f_k|^2 |g_k|^2} \cdot s + w_d + \rho \sum_{k=1}^{K} g_k w_k \qquad (3)$$

where w_d and w_k are noises at destination and relays. Observe that the channel is equivalent to an AWGN channel whose SNR at the destination is given by

$$\gamma_{ab} = \frac{\rho^2 P_1 \sum\limits_{k=1}^{K} |f_k|^2 |g_k|^2}{\sigma^2 + \rho^2 \sigma^2 \sum\limits_{k=1}^{K} |g_k|^2} \qquad (4)$$

III. MI of AF-DSTBC over the RSRC

It is well known that the capacity of an AWGN channel with complex codebook is given by [16]

$$C = \log(1 + \mathrm{SNR}) \quad \text{bit/dimension pair} \qquad (5)$$

By assuming perfect interleaving, the channel could be treated as an ergodic fading channel, whose average MI is shown in [17] to be the expectation of capacity of each channel realization. Denote $\bar{\mathcal{I}}$ as the average MI of the channel and $\bar{\mathcal{I}}_K$ as the average MI when there are K active relays. Then,

$$\bar{\mathcal{I}} = p_1(\xi, N)\bar{\mathcal{I}}_{K=1} + p_2(\xi, N)\bar{\mathcal{I}}_{K=2} + \cdots + p_N(\xi, N)\bar{\mathcal{I}}_{K=N} \qquad (6)$$

where

$$\bar{\mathcal{I}}_K = \mathrm{E}\left[\log\left(1 + \gamma_{ab}\right)\right] \qquad (7)$$

[1]Literature on the design of adequate DSTBC techniques for the random set relay channel is limited, but evidence on its feasibility can be found, *e.g.*, [4], [13]. GABBA codes [14], [15] are also promising candidates for such systems due to their arbitrary scalability.

When $K = 1$, it is derived in the appendix that $\bar{\mathcal{I}}_{K=1}$ is given by,

$$\bar{\mathcal{I}}_{K=1} = \frac{1}{\ln 2}\Bigg[e^{\varepsilon}\mathrm{Ei}\left(-\varepsilon\right) - e^{\frac{\varepsilon}{\bar{\gamma}_{ar}\xi+1}}\mathrm{Ei}\left(\frac{-\varepsilon}{\bar{\gamma}_{ar}\xi+1}\right) - e^{\frac{1+\bar{\gamma}_{ar}\xi}{\bar{\gamma}_{ar}}}\int_{0}^{\infty}\mathrm{Ei}\left(\frac{-\varepsilon-(\bar{\gamma}_{ar}\xi+1)y}{\bar{\gamma}_{ar}y}\right)e^{\frac{\varepsilon-\bar{\gamma}_{ar}y^2}{\bar{\gamma}_{ar}y}}\mathrm{d}y\Bigg] \quad (8)$$

where $\varepsilon \triangleq 1/\rho^2 = \frac{(1+\xi)P_1+\sigma^2}{P_2}$. When $K \geq 2$, it is impossible to get a simple closed form for $\bar{\mathcal{I}}_K$. However, we can use a simple upper bound to approximate it. Notice that function $y = \log(1+x)$ is concave, so by Jensen's Inequality [18]

$$\mathrm{E}\left[\log\left(1+\gamma_{ab}\right)\right] \leq \log\left(1+\mathrm{E}\left[\gamma_{ab}\right]\right) \quad (9)$$

Because f_k and g_k are independent and $\mathrm{E}\left[|f_k|^2\right] = 1+\xi$, we have

$$\mathrm{E}\left[\gamma_{ab}\right] = \frac{P_1(1+\xi)}{\sigma^2}\mathrm{E}\left[\frac{\sum_{k=1}^{K}|g_k|^2}{\varepsilon+\sum_{k=1}^{K}|g_k|^2}\right] \quad (10)$$

Notice that function $y = \dfrac{x}{a+x}$ is also concave for $a, x \geq 0$, so according to Jensen's Inequality:

$$\mathrm{E}\left[\gamma_{ab}\right] \leq \frac{P_1(1+\xi)K}{\sigma^2(\varepsilon+K)} \quad (11)$$

Putting (9) and (11) in (7), we have

$$\bar{\mathcal{I}}_K \leq \log\left(1+\frac{\gamma_{ar}K(1+\xi)}{K+\varepsilon}\right) \quad (12)$$

By law of large numbers, $\bar{\mathcal{I}}_K$ is asymptotically tight to the upper bound. It is shown by Monte Carlo simulation that it is tight enough even when K is not very large. So we use the upper bound to replace $\bar{\mathcal{I}}_K$ for $K \geq 2$. So far, we have all the materials to calculate $\bar{\mathcal{I}}$ using (6).

IV. RESULTS

In this section, we utilize (6) to investigate the impact of different parameter settings on the MI over the RSRC. Before that the tightness of the upper bound in (12) is shown via Monte Carlo simulation in figure 2 and 3. Observe that the difference between the two curves is 0.3 for $K = 3$, and decreases as K increases; the two curves have the same pattern which is another very important justification for utilizing the upper bound.

Then by setting the same transmit power constraint for each terminal, to be specific by setting $\gamma_{ar} = \gamma_{rb} = 10$dB, MI as a function of ξ for $N = 8, 12, 16$ is shown in figure 4. Notice that when $\xi = 0$ the transmission scheme is full-time-all-relay, so it is observed that by choosing a proper $\xi > 0$, there is always a gain in MI compared to full-time-all-relay scheme. This is because in AF scheme relays transmit amplified signals and noises simultaneously, which leads to that from each channel branch point of view the useful signal

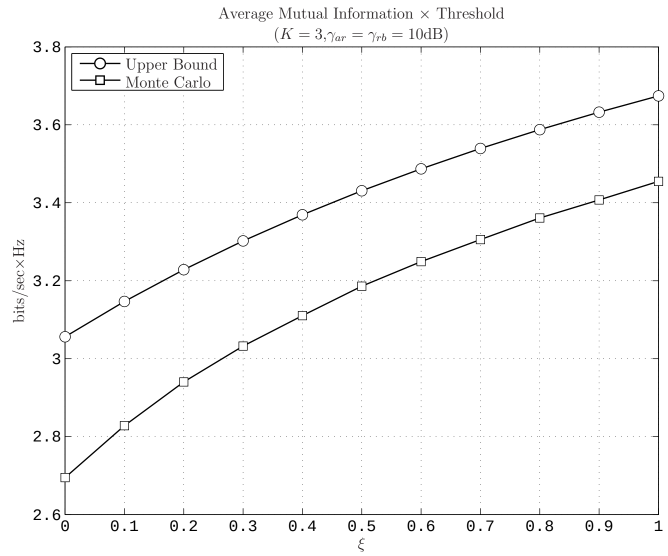


Fig. 2. Upper bound and Monte Carlo simulation for $K = 3$

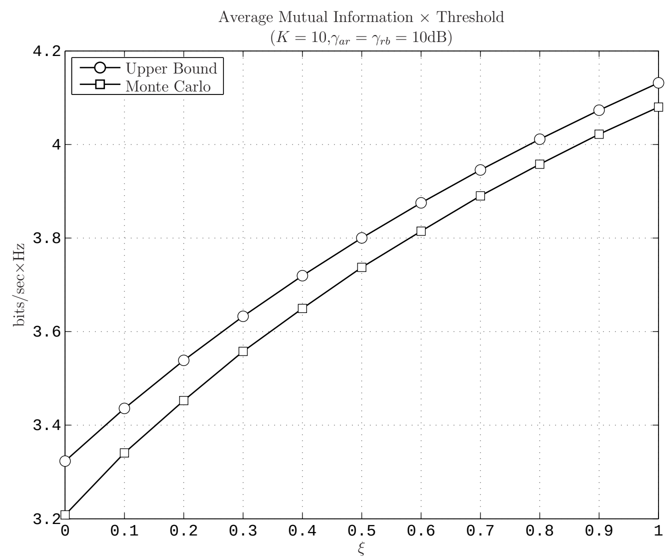


Fig. 3. Upper bound and Monte Carlo simulation for $K = 10$

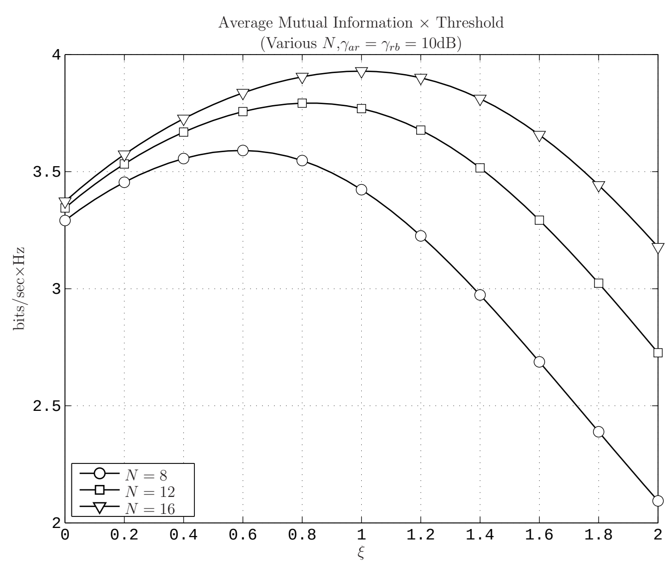


Fig. 4. MI as function of ξ parameterized by N

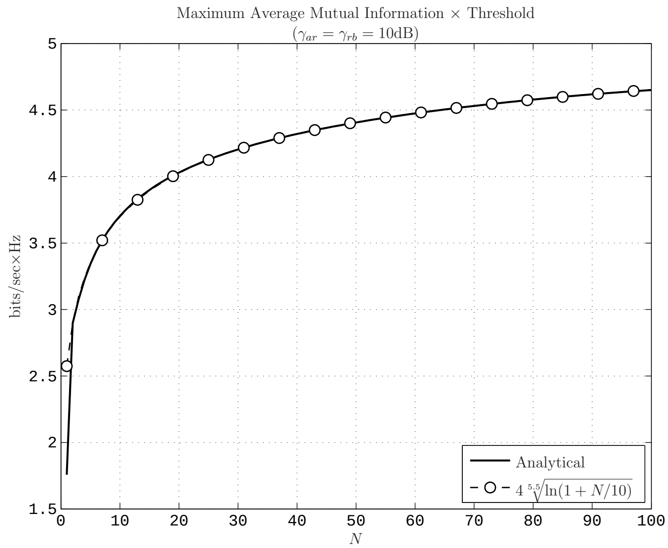


Fig. 5. MI as function of N parameterized by γ_{ar} and γ_{rb}

can only be degraded after each hop. As a result, It is no better to utilize a relay node in very bad backward channel state because no matter how much power is put on it, it could contribute mainly noise at the destination and degrade the overall performance. Meanwhile if ξ is set too large, the average active relay number could be too small and spacial diversity is lost. The larger N is, the more the gain is; The optimal MI increases with N which is intuitive. We are interested in how it increases with N and figure 5 shows that Optimal MI as function of N fits very well with function $4\sqrt[5.5]{\ln(1+N/10)}$ under the same setting: $\gamma_{ar} = \gamma_{rb} = 10$dB.

As mentioned, above analysis is based on equal power constraint per terminal, which leads to the fact that much more power is transmitted in the second stage than in the first stage when average active relay number is large. Inspired by the optimal power balance setting derived in [1], one may expect better energy efficiency if power balance between the two stages is more or less the same. So by setting $N = 8$ and fixing the total average transmit power $\gamma_{ab} = 13$dB, we study the impact of different power allocation on MI which is shown in figure 6 and 7. Here we define $\beta \triangleq \gamma_{ar}/\gamma_{ab}$ is the ratio of power consumed in the first stage to the overall transmitted power. It is observed that when $\xi = 0$, which corresponds to the full-time-all-relay scheme, the optimal power allocation is $\beta = 1/2$, which accords with the result in [1]; the optimal MI is obtained when $\xi = 0.8$ and $\beta \approx 0.4$; As ξ becomes larger, received signals at relays become more reliable, so more power could be allocated at the second transmission stage.

Finally, the mutual information over uncoded M-ary PSK and QAM signals, with block-wise hard-decision can be found by considering the binary symmetric channel with the transition probabilities $\bar{P}_b$ and $1-\bar{P}_b$ which are derived in [7]. It can be calculated by [18]

$$C_M(\gamma)=\log_2(M)\big[1+\bar{P}_b\cdot\log_2\bar{P}_b+(1-\bar{P}_b)\cdot\log_2(1-\bar{P}_b)\big] \quad (13)$$

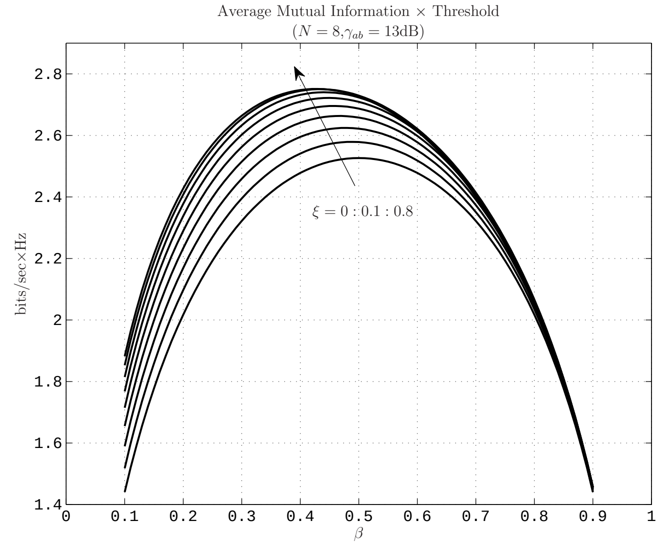


Fig. 6. MI as function of β parameterized by $\xi \leq 0.8$

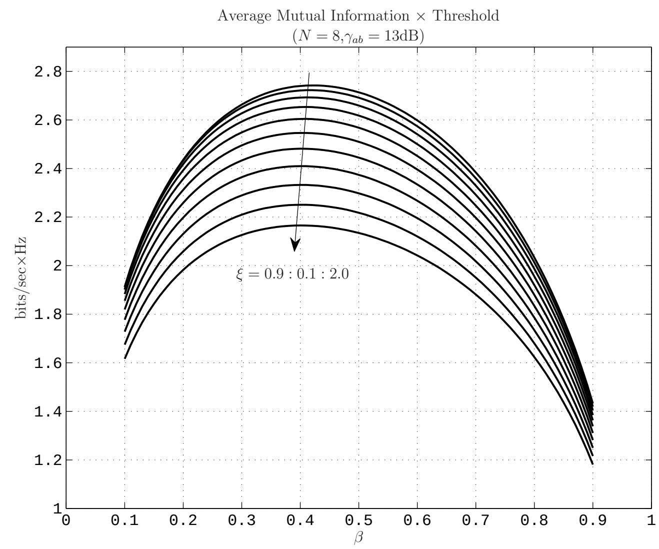


Fig. 7. MI as function of β parameterized by $\xi > 0.8$

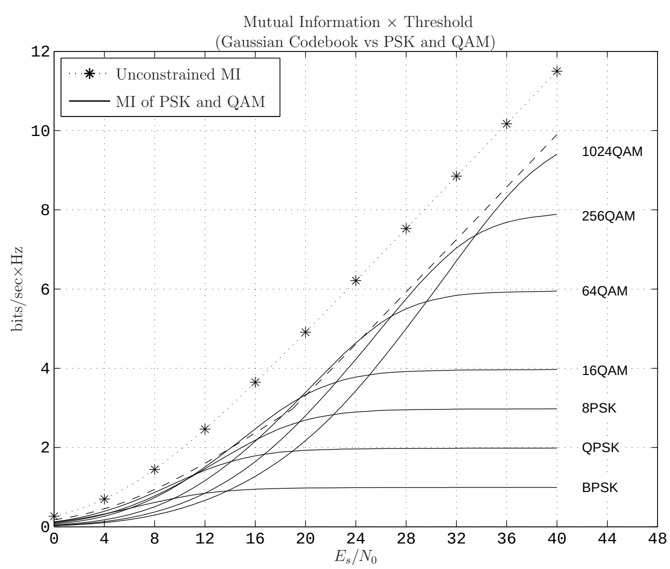


Fig. 8. Unconstrained and constrained MI against signal-to-noise ratio

Both unconstrained MI and MI over several PSK and QAM modulations are shown in figure 8. As can be observed that at high signal-to-noise ratio region, the optimal PSK-or-QAM modulated MI comes with nearly 2 bits loss compared to the unconstrained MI.

V. Conclusion

We studied the performance of ideal AF-DSTBCs over the RSRC which is the result of an autonomic backward-CSI-based relay selection scheme. Mutual information over the resulting RSRC were derived. Using the expressions, it was shown that such systems can achieve substantial gains over equivalent AF-STBCs systems where a fixed number (all) of relays are active at all times. Meanwhile, it was shown that generally the optimal power balance between the two transmission stages is not 1:1; with a properly chosen ξ more power should be put in the second stage.

Appendix

Derivation of $\bar{\mathcal{I}}_{K=1}$ will be shown in the camera ready version of this manuscript.

References

[1] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

[2] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," in *Proc. IEEE Int. Conf. of Acoustics, Speech, and Sig. Proc.(ICASSP'07)*, Hawaii, USA, Apr. 2007.

[3] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Commun.*, vol. 6, no. 9, pp. 3450–3460, 2007.

[4] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[5] B. Maham, A. Hjorungnes, and G. Abreu, "Distributed GABBA space-time codes in amplify-and-forward relay networks," in *IEEE Sensor Array and Multichannel Signal Processing Workshop(SAM'08)*, Darmstadt, Germany, Jul.21-23 2008.

[6] ——, "Distributed GABBA space-time codes in amplify-and-forward relay networks," *IEEE Trans. Wireless Commun.*, 2008, (submitted, apr).

[7] Q. Xue, G. Abreu, and B. Aazhang, "Analysis of amplify-and-forward DSTBCs over the random set relay channel," in *Forty-Two Asilomar Conference on Signals, Systems and Computers*, Monterey, CA, Oct. 2008.

[8] R. J. McEliece and W. E. Stark, "Channels with block interference," *IEEE Trans. Inf. Theory*, vol. IT-30, pp. 44–53, Jan. 1984.

[9] B. Wang, Z. Han, and K. J. R. Liu, "Distributed relay selection and power control for multiuser cooperative communication networks using buyer-seller game," in *Proc. IEEE 26rd Annual Conference Conference on Computer Communications (Infocom'07)*, Anchorage, USA, Mar.6-12 2007, pp. 544 – 552.

[10] S. Barbarossa, L. Pescosolido, D. Ludovici, L. Barbetta, and G. Scutari, "Cooperative wireless networks based on distributed space-time coding," in *Proc. IEEE International Workshop on Wireless Ad-hoc Networks (IWWAN'04)*, Oulu, Finland, May31Jun.3 2004.

[11] A. K. Sadek, W. Su, and K. J. R. Liu, "Clustered cooperative communications in wireless networks," in *IEEE Global Conference on Communications (Globecomm'05)*, St. Louis, U.S.A., Nov.28Dec.2 2005.

[12] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 235–237, Apr. 2008.

[13] B. S. Mergen and A. Scaglione, "Randomized distributed space-time coding for cooperative communication in self-organized networks," in *Proc. IEEE Workshop on Signal Process, Advances in Wireless Commun. (SPAWC'05)*, NY, USA, Jun.5-8 2005.

[14] G. T. F. de Abreu, "GABBA codes: Generalized full-rate orthogonally decodable space-time block codes," in *Proc. IEEE The 39-th Asilomar Conference on Signal, Systems and Computers (ASILOMAR'05)*, Monterey, USA, Oct 28 - Nov 1 2005.

[15] ——. (2005) Generalized ABBA space-time block codes. Available at ⟨http://arxiv.org/abs/cs/0510003v1⟩.

[16] E. Biglieri, *Coding for wireless channels*, 1st ed. Springer, 2005.

[17] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 9, pp. 1986–1992, Nov. 1997.

[18] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Wiley-Interscience, 2006.