Multi-Hop Aggregate Information Efficiency in Wireless Ad Hoc Networks

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***Abstract*—We introduce *multi-hop aggregate information efficiency* ($\mathrm{MIE_A}$), a comprehensive metric that captures several performance-affecting factors of wireless ad hoc networks in a unified formulation. This metric is then employed to analyze such networks with respect to their spectral efficiencies, network loads, and hopping strategies. The analysis reveals that the hopping strategy that achieves maximum information efficiency is that of multiple short hops with no more than a single packet retransmission allowed at each hop, as opposed to the alternative of fewer long-haul hops with multiple packet retransmissions. The implementation of that preferred strategy withstanding, it is found furthermore that the most efficient networks typically exhibit about 65% of link outage probability, which corroborates similar findings obtained in different network settings and using different metrics. Bearing in mind that link outage is a function not only of deterministic parameters such as node density, but also of design parameters such as modulation, our analysis also shows that the modulation scheme that optimizes the aggregate information efficiency is in fact a function of node density. In that respect, our metric and method is shown to be useful to determining the modulation scheme that optimizes the performance of a network with a certain node density.**

I. Introduction

Due to the absence of coordination and their inherently random nature wireless ad hoc networks are difficult to analyze. Unlike peer-to-peer wireless communication systems, for which an array of well-established performance evaluation metrics exists, part of the problem of characterizing the performance of wireless ad hoc networks is to establish metrics adequate for that purpose. In addressing this problem, a handful of performance measures have been proposed to capture the peculiarities of these challenging networks.

In [1], for instance, Kumar and Gupta introduced the *transport capacity*, defined as the number of bit-meters that flows throughout the network in a given time interval. The model thereby, however, assumes a deterministic signal-to-interference ratio (SIR), which implies that the occurrence of outage events is mitigated by the medium access control (MAC) mechanism.

In [2], Sousa and Silvester obtained the best choice of the transmission range in a multi-hop scenario, based on the so-called *expected forward progress*, which measures the average distance traveled by a packet in each hop.

The expected forward progress formulation was then extended by Subbarao and Hughes [3] so as to account for the gains related to a more efficient spectrum usage, which led to an yet better metric dubbed the *information efficiency* (IE), defined as the product of the expected forward progress and spectral efficiency of the transmission system.

Although the information efficiency captures several aspects of wireless ad hoc networks, it does not take into account the spatial reuse of the radio channel. In order to capture this important additional factor, Mignaco and Cardieri [4] extended the concept of information efficiency and proposed the *aggregate information efficiency* ($\mathrm{IE_A}$), defined as the sum of the amount of information efficiency of all single-hop active links of the network at a given time, normalized by the network area. The aggregate information efficiency was then employed in [5] to study the effects of some transmission system parameters on the performance of wireless ad hoc networks.

A limitation that remained in the aforementioned metrics was that link outage was considered to be a predictable event, when in fact outage has a stochastic behavior, due to the random characteristics of the network dynamics and signal variations. In [6], Weber *et al.* proposed to mitigate that limitation by including an outage constraint and the spectral efficiency of the transmission system into the network analysis, obtaining a new network performance measure referred to as the *transmission capacity*. This metric has been applied in other works [7] [8] to study the performance of several techniques used to improve the capacity of wireless systems.

Inspired by those works, Nardelli and Cardieri [9] finally proposed a modified definition of the aggregate information efficiency by including an outage probability into formulation.

The contributions of this work is as follows. First, we propose a generalization of the $\mathrm{IE_A}$ that is better suitable to address multi-hop networks. This is achieved by abstracting the number of hops of multi-hop links and characterizing them simply via their higher packet loss probabilities and lower spectral efficiencies, yielding a metric referred to as the *multi-hop aggregate information efficiency* ($\mathrm{MIE_A}$). Then, we study the network configurations that maximize the $\mathrm{MIE_A}$, particularly with respect to the interrelations between the number of hops n_h, the average outage probability $\bar{P}_\mathrm{out}$, the average density of active links $\bar{\rho}_\mathrm{at}$, the average single-hop distance $\bar{d}_\mathrm{tr}$ and robustness, evaluated in terms of the minimum SIR required to guarantee a certain packet loss probability.

The remainder of the paper is organized as follows. The concept of multi-hop aggregate information efficiency is introduced in Section II. The characterization of the network behavior is presented in Section III, followed by the system description in Section IV. Numerical results are analyzed in Section V, and conclusions are given in Section VI, along with ideas for possible future work.

II. Multi-Hop Aggregate Efficiency Information

In order to evaluate how efficient is the information flow through a single-hop link, Subbarao and Hughes proposed in [3] the concept of *information efficiency* (IE), defined as

$$\mathrm{IE} = \bar{d}_{\mathrm{tr}} \times (1 - P_{\mathrm{sys}}) \times \eta_{\mathrm{sys}}, \tag{1}$$

where $\bar{d}_{\mathrm{tr}}$ is the average distance between transmitter and receiver, P_{sys} is the packet loss probability of a link, and η_{sys} is the spectral efficiency of the radio interface.

However, the IE does not consider the spatial reuse of the channel and, consequently, the co-channel interference. In order to incorporate this relevant aspect, Mignaco and Cardieri [4] proposed the *aggregate information efficiency* ($\mathrm{IE_A}$), measured in bits per Hz-meter-second and defined by

$$\mathrm{IE_A} = \bar{d}_{\mathrm{tr}} \times (1 - P_{\mathrm{sys}}) \times \eta_{\mathrm{sys}} \times \bar{\rho}_{\mathrm{at}} \times (1 - \bar{P}_{\mathrm{out}}), \tag{2}$$

where ρ_{at} is the average density of active links in the network and $\bar{P}_{\mathrm{out}}$ is the average link outage probability.

Notice that the $\mathrm{IE_A}$ is in fact the sum of IE of all single-hop links active in the network at a given time, normalized by the network area, since $\rho_{\mathrm{at}} \times (1 - \bar{P}_{\mathrm{out}})$ is indeed the density of active links that satisfy the quality of service requirement.

Although this metric is suitable to study networks composed by single-hop links, the $\mathrm{IE_A}$ formulation does not capture directly the multi-hop case. In order to mitigate this limitation we propose the *multi-hop aggregate information efficiency* ($\mathrm{MIE_A}$), defined as

$$\mathrm{MIE_A} = d_{\mathrm{sd}} \times (1 - P_{\mathrm{mh}}) \times \eta_{\mathrm{mh}} \times \bar{\rho}_{\mathrm{at}} \times (1 - \bar{P}_{\mathrm{out}}), \tag{3}$$

where d_{sd} is the average distance from all source-destination pairs, and P_{mh} and η_{mh} are the packet loss probability and the spectral efficiency of multi-hop links.

As indicated by equation (3), the formulations of the $\mathrm{IE_A}$ and $\mathrm{MIE_A}$ are similar. The essence of the extension is to abstract the number of hops of multi-hop links and characterize such links simply via their higher packet loss probabilities and lower spectral efficiencies. In other words, the parameters d_{sd}, P_{mh} and η_{mh} used in equation (3) must incorporate the impact of multi-hopping onto the distance, the packet loss probability and the spectral efficiency of source-to-destination links.

Employing a 2-dimensional toroidal square model with sides of length L (such that all nodes can be analyzed as the center of the network), and under the assumptions of uniformly randomly distributed nodes with random formation of source-destination pairs, d_{sd} can be computed by

$$d_{\mathrm{sd}} = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{1}{L^2} \sqrt{x^2 + y^2}\, dx dy = 0.3826 \times L. \tag{4}$$

Under the assumption that the probability of successful packet transmission is independent at each hop, the packet loss probability of a n_{h}-hop link is simply

$$P_{\mathrm{mh}} = 1 - (1 - P_{\mathrm{sys}})^{n_{\mathrm{h}}}. \tag{5}$$

Finally, since a packet traveling over an n_{h}-hop link requires n_{h} times as much channel uses than a single-hop link, the overall spectral efficiency a multi-hop link is

$$\eta_{\mathrm{mh}} = \frac{\eta_{\mathrm{sys}}}{n_{\mathrm{h}}}. \tag{6}$$

Except for these three new equations, the $\mathrm{MIE_A}$ requires the same parameters as the $\mathrm{IE_A}$ to be computed, namely, the systemic parameters η_{sys} and P_{sys}, and the network behavioral parameters $\bar{P}_{\mathrm{out}}$, $\bar{\rho}_{\mathrm{at}}$ and $\bar{d}_{\mathrm{tr}}$. These parameters will be determined in the sequel.

III. Behavioral Parameters $\bar{P}_{\mathrm{out}}$, $\bar{\rho}_{\mathrm{at}}$ and $\bar{d}_{\mathrm{tr}}$

A. Network Model and Characterization

Consider a wireless network with nodes randomly placed in a toroidal square region of area A m^2 and density ρ nodes/m^2. Assume that all nodes are equipped with omnidirectional antennas and transmissions occur in a time-slot basis and that the final destination of each generated packet is chosen randomly with uniform (equal) probability amongst the other nodes in the network area.

Let α be the distance-dependent path loss exponent and H the random gain (independent of the distance) related to the fading phenomenon, modeled here as a standard log-normal distribution [10].

Assuming that all nodes have the same transmit power W_{t}, and employing a bounded receive power model [10], the received power W_{r} is given by

$$W_{\mathrm{r}} = \begin{cases} \dfrac{HW_{\mathrm{t}}}{d_{\mathrm{tr}}^{\alpha}} & \text{if} \quad d_{\mathrm{tr}} \geq d_0 \\ HW_{\mathrm{t}} & \text{if} \quad d_{\mathrm{tr}} < d_0, \end{cases} \tag{7}$$

where d_0 is the reference distance (fixed here to one meter).

It is assumed that a receiver is able to receive satisfactorily a packet transmitted by a given node if: a) the received power is higher than a given minimum W_{min}, and b) the signal-to-interference ratio at the receiver is higher than a threshold $\mathrm{SIR_{th}}$. Thus, the SIR at a node j is given by

$$\mathrm{SIR}_j = \frac{W_j}{\sum\limits_{i \in \mathcal{I}} W_i}, \tag{8}$$

where $\mathcal{I}$ is the set of interferers to node j, and where W_j and W_i are the desired receive power and the interference powers, respectively.

B. Computation of $\bar{P}_{\mathrm{out}}$, $\bar{\rho}_{\mathrm{at}}$ and $\bar{d}_{\mathrm{tr}}$

Given the above-described parameters, the link outage probability $\bar{P}_{\mathrm{out}}$, the average density of active links $\bar{\rho}_{\mathrm{at}}$ and the average single-hop distances $\bar{d}_{\mathrm{tr}}$ can be computed via the Monte Carlo simulation described by the following steps:

1) Generate a set of $\rho \times A$ nodes randomly distributed over a 2-dimensional toroidal square of area A m^2.
2) Generate random, log-normally distributed channel gains for all node pairs of the network.
3) Identify the neighbors of each node as those whose receive power (considering both the channel gain and path loss) are higher than the minimum power W_{min} required for packet reception.
4) Select at random any node that has neither been labeled inactive nor formed an active link to another node.
5) Assign to the selected node its most distant neighbor available as its *receive node*. If no receiving node can be assigned[1], label the selected node *inactive*. Otherwise label the selected node a *transmit node* and the link between the transmit and receive an *active link*.
6) Repeat steps 4 and 5 until all nodes have been labeled transmit, receive or inactive.
7) Count the number of active links N_{at} and compute the density of active links in the current network realization (snapshot) as $\rho_{\text{at}} = N_{\text{at}}/A$.
8) Compute the SIR at each receive node. If SIR $\leq$ SIR$_{\text{th}}$ at a given node, label such a link as in *outage*.
9) Count the number of outage links N_{out} and determine the outage probability for the snapshot by $P_{\text{out}} = N_{\text{out}}/N_{\text{at}}$.
10) Compute the average distance of all active links not in outage d_{tr} for the snapshot.
11) Repeat the steps 1 through 10 several times and average P_{out}, ρ_{at} and d_{tr} over all snapshots, obtaining $\bar{P}_{\text{out}}$, $\bar{\rho}_{\text{at}}$ and $\bar{d}_{\text{tr}}$.

Figures 1 to 3 show a numerical results obtained with this simulation procedure for a few different values of W_{min} and with $\alpha = 4$ and SIR$_{\text{th}} = 100$. It can be seen that the average density of active links $\bar{\rho}_{\text{at}}$ increases with the node density ρ. This is non-surprising since in network scenario under consideration both the traffic and the probability of forming active links are directly proportional to the density of nodes in the network. Consequently, it is observed that the average outage probability $\bar{P}_{\text{out}}$, which depends on the average number of active (interfering) links, also increases with the node density. Likewise, it is found that the average single-hop link distance $\bar{d}_{\text{tr}}$ grows in proportion to ρ, since the larger the node density, the higher the probability that a node finds a distant neighbor to pair up with.

Finally, all the aforementioned parameters are found to decrease with the minimum receive power required for successful packet reception W_{min}. This is because larger W_{min} effectively reduces the neighborhood vicinity of all nodes, and with a given ρ this implies a smaller number of neighbors available for pairing, with consequent lower $\bar{\rho}_{\text{at}}$, $\bar{P}_{\text{out}}$ and $\bar{d}_{\text{tr}}$.

Notice the above-described Monte Carlo simulation requires the parameter SIR$_{\text{th}}$ as an input. The computation of SIR$_{\text{th}}$, as well as the other systemic parameters P_{sys} and η_{sys} are discussed in the next subsection.

[1]Notice that some nodes may remain unpaired, either because all their neighbors have been already paired with other nodes, or because they have no neighbors at all.

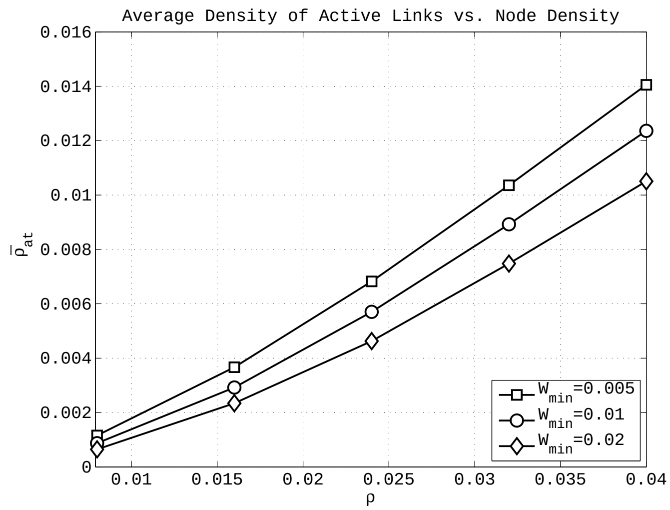

Fig. 1. Average density of active links $\bar{\rho}_{\text{at}}$ as a function of the density of nodes ρ, parameterized by W_{min}, with $\alpha = 4$ and SIR$_{\text{th}} = 100$.

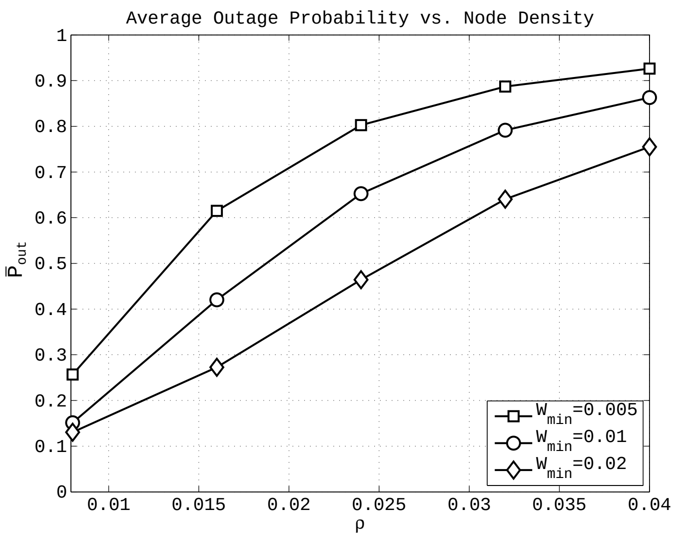

Fig. 2. Average outage probability $\bar{P}_{\text{out}}$ as a function of the density of nodes ρ, parameterized by W_{min}, with $\alpha = 4$ and SIR$_{\text{th}} = 100$.

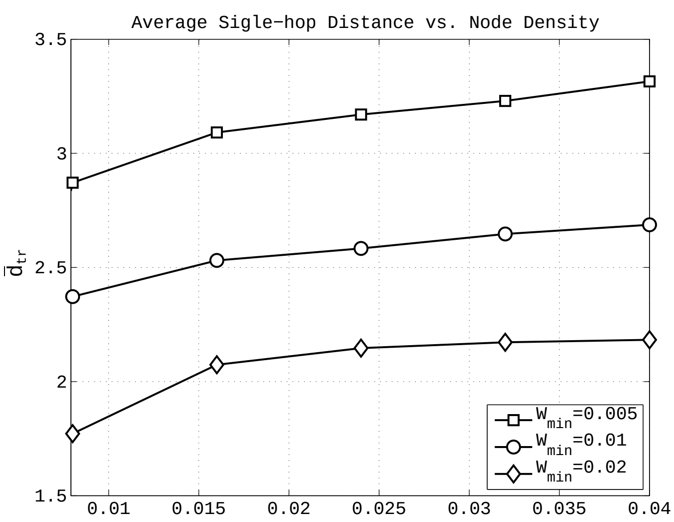

Fig. 3. Average transmitter-receiver distance $\bar{d}_{\text{tr}}$ as a function of the density of nodes ρ, parameterized by W_{min}, with $\alpha = 4$ and SIR$_{\text{th}} = 100$.

IV. Systemic Parameters P_{sys}, η_{sys} and the Threshold SIR

A. The Packet Loss Probability P_{sys}

Consider a *selective-repeat* automatic repeat request (ARQ) retransmission scheme [11] represented by the state diagram shown in figure 4, which is employed at each hop of a multi-hop packet forwarding procedure. In such a scheme, a node attempts up to m retransmissions of a packet, each subject to an independent packet error probability P_{pct}.

The reception of each packet is acknowledged either with an ACK, in case of successful decoding, or a NACK, in case a packet error has been detected. Notice that links in outage are already accounted for by the network simulation procedure described in the preceding section, such that no other possibilities rather than packets acknowledged by an ACK or NACK may occur over active links.

A packet is lost if the m-th packet retransmission fails, $i.e.$, $m+1$ NACKs have been detected by the transmitter. Let P_m denote the probability that a packet has been unsuccessfully transmitted m times, such that the node allocates it to a last retransmission. The packet loss probability P_{sys} is given by the packet error probability at the last retransmission. Thus,

$$P_{\text{sys}} = P_{\text{pct}} \cdot P_m = P_{\text{pct}}^{\,m+1} \frac{1-P_{\text{pct}}}{1-P_{\text{pct}}^{\,m+1}}, \tag{9}$$

where P_m is computed using standard queue algebra as described below.

Let a packet that has been retransmitted q times be referred to as being *in state* q. Referring to figure 4, the probability that a packet in state 0 progresses to state q (after q unsuccessful retransmissions) is given by

$$P_q = P_0 \cdot P_{\text{pct}}^{\,q}. \tag{10}$$

The probability P_0 that a packet is in state 0 can be computed, using equation (10) and $\sum_{q=0}^{m} P_q = 1$, yielding

$$P_0 = \frac{1-P_{\text{pct}}}{1-P_{\text{pct}}^{\,m+1}}. \tag{11}$$

Finally, substituting equation (11) into (10) with $q = m$ finally yields P_m as in equation (9).

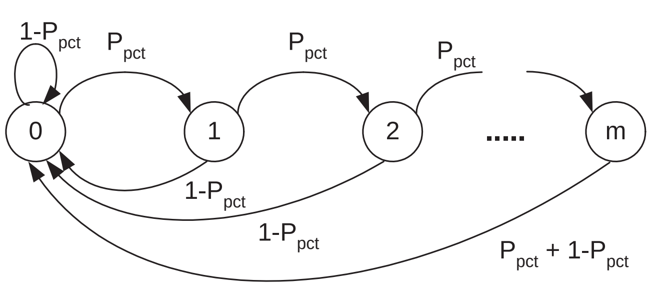

Fig. 4. Diagram of the selective-repeat ARQ retransmission scheme [11]. A packet can be retransmitted at most m times. The probability of an error in the packet reception is P_{pct}. A packet is lost if a reception error occurs after m unsuccessful trials.

Next, we must compute the packet error probability P_{pct}, which in turn depends on the modulation and coding schemes employed. We follow [3] and consider a system employing M-ary QAM modulation and an extended (M, k) Reed-Solomon code [12] with rate k/M and block length M equals to the cardinality of the modulation alphabet, which is capable of correcting up to $\frac{M-k}{2}$ symbol errors in a code-block.

With these choices of coding and modulation[2], and assuming that interference dominates over additive noise and can be modeled as Gaussian random variates [2], the codeword and symbol error probabilities are respectively given by

$$P_{\text{cod}} = \sum_{n=\frac{M-k}{2}+1}^{M} \binom{M}{n} P_{\text{mod}}^{\,n} (1-P_{\text{mod}})^{M-n}, \tag{12}$$

$$P_{\text{mod}} = 2\left(1-\frac{1}{\sqrt{M}}\right) \text{erfc}\left(\sqrt{\frac{3}{2}\frac{\log_2 M}{(M-1)}\frac{\text{SIR}}{\eta_{\text{mod}}}}\right), \tag{13}$$

where η_{mod} is the spectral efficiency of M-QAM [13]

$$\eta_{\text{mod}} = \log_2 M. \tag{14}$$

Finally, assuming a packet format of B bits, such that each packet carries $n_{\text{c}} = B/(k \log_2 M)$ codewords, and since a packet is successfully received if all its codewords are correctly decoded, the packet error probability P_{pct} becomes

$$P_{\text{pct}} = 1-(1-P_{\text{cod}})^{n_{\text{c}}}. \tag{15}$$

B. The Spectral Efficiency η_{sys}

The spectral efficiency η_{sys} of the transmission system described above is given by

$$\eta_{\text{sys}} = \log_2 M \times \frac{k}{M} \times \frac{1}{\bar{q}}, \tag{16}$$

where $\bar{q}$ denotes the average number of times a packet is transmitted when hopping from a node to another, which is

$$\bar{q} = \sum_{q=0}^{m} (q+1) P_q = \frac{1-P_{\text{pct}}}{1-(P_{\text{pct}})^{m+1}} \times \tag{17}$$
$$\times \left(\frac{(1-(P_{\text{pct}})^{m+1}) - ((m+1)(P_{\text{pct}})^{m+1}(1-P_{\text{pct}}))}{(1-P_{\text{pct}})^2}\right).$$

C. The Threshold Signal-to-interference-ratio SIR_{th}

The quality of source-to-destination links in a multi-hop ad hoc network depends inevitably on both the expected number of hops and the expected packet loss probability of the link, in the sense that paths requiring more hops will in average be worse, that is, have a higher P_{mh}, than paths with fewer hops. Assuming that these expectations (n_{h} and P_{mh}) are given, one can compute the maximum average packet error probability P_{sys} admissible at each hop, and corresponding threshold SIR_{th}, using equation (5) and the set of sequence of equations (9)→(15)→(12)→(13), respectively, yielding

$$\text{SIR}_{\text{th}} = \frac{2}{3}\left(\frac{(M-1)\eta_{\text{mod}}}{\log_2 M}\right) \text{erfc}^{-1}\left(\frac{P_{\text{mod}}\sqrt{M}}{2(\sqrt{M}-1)}\right). \tag{18}$$

[2]Notice that this physical-layer specification play no fundamental role in the network analysis. Other choices of coding and modulation could also be made with similar overall results.

V. Analysis of ad hoc Networks via MIE_A

Using the outcomes of sections III and IV, we are almost fully instrumented to evaluate the MIE_A defined in equation (3). In particular, substituting equations (4) and (5) we have

$$\text{MIE}_\text{A} = \frac{d_\text{sd}}{n_\text{h}} \times (1 - P_\text{sys})^{n_\text{h}} \times \eta_\text{sys} \times \bar{\rho}_\text{at} \times (1 - \bar{P}_\text{out}), \quad (19)$$

where the computation of all but the parameter n_h have been addressed in sections II, III and IV.

The typical number of hops n_h in an ad hoc network depends on several other factors beyond the scope of this article, including the routing mechanism, the application, etc. In any case, an optimistic estimate is

$$n_\text{h} = \frac{d_\text{sd}}{\bar{d}_\text{tr}}. \quad (20)$$

In average terms, this represents an ideal hopping pattern where the average source-to-destination distance d_sd is travelled in a straight line and in leaps of $\bar{d}_\text{tr}$, which is the average single-hop distance of the network. In other words, this implies an assumption that the routes from sources to destinations have negligible deviation. Consequently, the MIE_A results shown hereafter, obtained using equation (20) into (19) can be seen as an upper bound on the multi-hop aggregate information efficiency.

The objective of our analysis is to evaluate how the maximum number of retransmission m, the minimum received power W_min and the modulation order M affect the MIE_A, considering a network specified by Table I .

Figure 5 presents the multi-hop aggregate information efficiency as a function of the maximum number of retransmissions m for a few different values of W_min and 32-QAM modulation scheme. The results show that $m = 1$ achieves the maximum MIE_A, regardless of W_min. The choice of m has an evident influence on the network interference robustness. Specifically, the system characterization (detailed in Section IV) shows that larger values of m lead to lower thresholds SIR_th required to guarantee the desired packet loss probability P_sys. As a direct consequence of lower thresholds SIR_th, the average outage probability $\bar{P}_\text{out}$ diminishes. However, larger m have a negative impact on the system efficiency (if more retransmissions are allowed, higher P_pkt are acceptable and η_sys reduces). Therefore, there is a trade-off related to the effects of the maximum number of retransmissions on the MIE_A and $m = 1$ indicates the best choice to optimize this trade-off[3].

In addition to this aspect, Figure 5 also reveals that higher minimum received powers required W_min have higher MIE_A. Since the average transmitter-receiver distance $\bar{d}_\text{tr}$ decreases as W_min increases, packets will travel (in average) over a greater number of single-hops to achieve their destinations (n_h increases). Then, the probability of a packet loss in a multi-hop link P_mh increases. Moreover, higher W_min implies a lower

[3]It is important to emphasize that similar results have been obtained in previous works [5] [9], although the network modeling used here is quite different.

TABLE I
Network specification

Parameter	Specification
Network Area	$A = 50 \times 50\ m^2$
Path loss exponent	$\alpha = 4$
Max. packet loss probability	$P_\text{sys} = 10^{-3}$
Modulation	M-ary QAM
Code Rate	$k/M = 0.75$
Packet length	$B = 1600$ bits

average density of active links $\bar{\rho}_\text{at}$. On the other hand, the increase of W_min diminishes the outage occurrence. Considering the aforementioned aspects, it is possible to conclude that it is better to have multi-hop links composed by a larger number of more robust single-hop links. It is interesting to notice that this result indicates that the hopping strategy of choosing as a receiver the most distant neighbor available could be the wrong strategy to optimize the MIE_A.

Now, the effects of the modulation scheme on the MIE_A are studied. Figure 6 shows the multi-hop aggregate information efficiency as a function of the modulation order M for different node density ρ, and setting $W_\text{min} = 0.01$ and $m = 1$. The results show that the modulation order that maximizes the MIE_A depends on the network node density ρ. The effects of the higher order modulations are two-fold. On the one hand, the transmissions are more efficient (higher η_sys). On the other hand, a higher SIR_th is required to achieve the maximum packet loss probability P_sys. Therefore, for the same node density, the maximum value MIE_A reflects the best trade-off between those aspects.

In order to study the effects of the network node density on the MIE_A, it is useful to know how the behavioral parameter $\bar{P}_\text{out}$ affects the network performance. As an illustrative example, Figure 7 presents the MIE_A as a function of the average outage probability $\bar{P}_\text{out}$ for 32-QAM and 128-QAM, and setting $W_\text{min} = 0.01$ and $m = 1$. Firstly, one can notice that there is a value of $\bar{P}_\text{out}$ that maximizes the MIE_A. This value is around 65% for both modulation schemes, and it reflects the optimal trade-off between node activity and outage events (more active links imply higher interference level). In addition, Figure 7 shows that 32-QAM modulation performs better than 128-QAM if it is considered the same $\bar{P}_\text{out}$,. However, $\bar{P}_\text{out}$ is not an input parameter (indeed, $\bar{P}_\text{out}$ is a parameter that reflects the network interference susceptibility). For an equal node density, one can verify that $\bar{P}_\text{out}$ is different for each modulation. Based on this fact, the results shown in Figure 6 can be explained as follows. The increase of the node density leads to higher interference level, that corresponds a higher $\bar{P}_\text{out}$. As commented above, there is a value of $\bar{P}_\text{out}$ that maximizes the MIE_A, and the density ρ related to this point depends on the modulation order M. Finally, it is possible to conclude that the results presented in Figure 6 reflects the effects of the modulation order on η_sys and SIR_th, associated with the trade-off between ρ_at and $\bar{P}_\text{out}$.

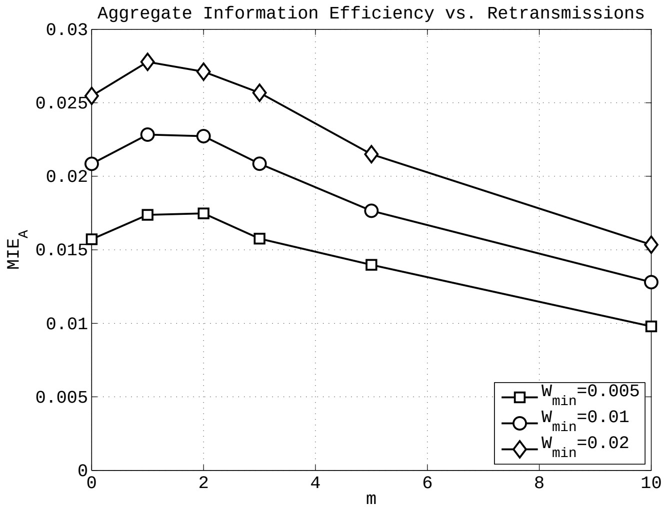

Fig. 5. $\mathrm{MIE_A}$ as a function of the maximum number of retransmissions m with $\rho = 0.04$, parameterized by $W_{\min}$, and using 32-QAM modulation scheme.

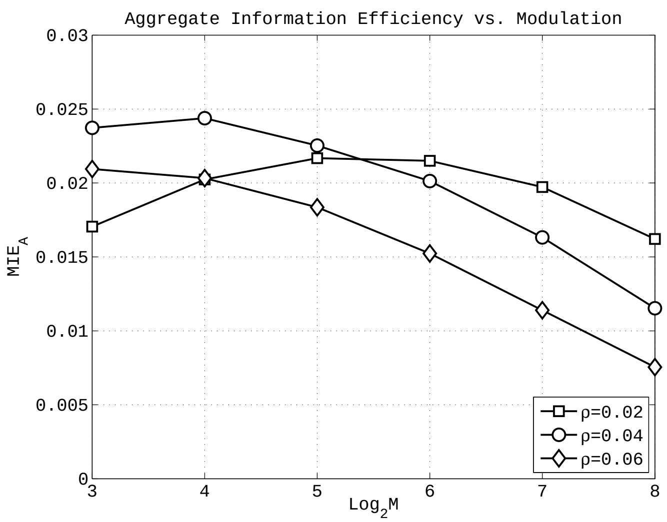

Fig. 6. $\mathrm{MIE_A}$ as a function of the modulation order M for different node densities ρ, and setting $W_{\min} = 0.01$ and $m = 1$.

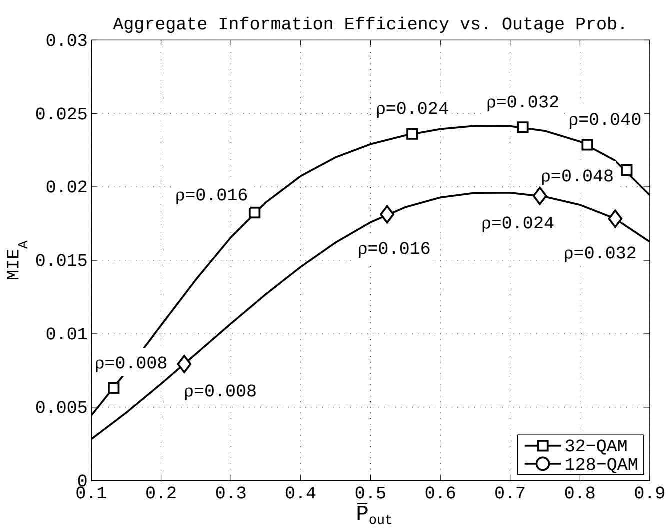

Fig. 7. the $\mathrm{MIE_A}$ as a function of the average outage probability $\bar{P}_{\text{out}}$ for 32-QAM and 128-QAM, and setting $W_{\min} = 0.01$ and $m = 1$

VI. Conclusion and Future Works

In this paper we proposed the multi-hop aggregate information efficiency, a suitable metric to evaluate the performance of multi-hop wireless networks in a simple formulation. Using this metric, the effects of the maximum number of retransmissions allowed (for packets detected in error), the hopping strategy and the modulation order on the network performance have been studied.

The results indicate that a strategy of multiple short single-hops using a single packet retransmission achieves the highest $\mathrm{MIE_A}$. In addition, it is shown that the selection of the modulation scheme to optimize the $\mathrm{MIE_A}$ depends on the network node density. Finally, it is found that the average outage probability that maximizes the $\mathrm{MIE_A}$ is about 65%. In this case, the information flow throughout the network is maximized, although the number of active links in outage situation increases.

The concepts and results presented here give us some directions for future works. For example, compare the performance of routing algorithms, redo the network evaluation for different policies of relay selection (closest neighbor or random neighbor) and study the performance of an adaptive modulation scheme to improve the $\mathrm{MIE_A}$.

Furthermore, we intend to model the network behavior using the tools of stochastic geometry, random graph and percolation theories. Based on this analytical approach, it will be possible to suppress the Monte-Carlo simulation used in this present work.

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