Modified Orthogonal Space-Time Block Codes for Time-Selective Fading Channels

Gabriel P. Villardi, *Student Member, IEEE,* Giuseppe T. F. de Abreu, *Member, IEEE,* and Ryuji Kohno, *Member, IEEE*

***Abstract*— We consider orthogonal space-time block codes (O-STBCs) in time-selective fading channels and propose a modified O-STBC (MO-STBC), which is less subject to performance degradation caused by symbol-by-symbol channel variation. It is shown both analytically and through computer simulations that MO-STBCs effectively reduce the loss of orthogonality affecting O-STBCs in the symbol-by-symbol time-selective channel. MO-STBCs are systematically constructible for any number of antennas, and have the same decoding complexity of conventional O-STBCs. The bit-error rate performance of MO-STBCs over time-selective fading channels of generalized statistics, with Gray code mapped M-ary PSK and QAM modulations, is analytically derived. Moreover, MO-STBCs can also be easily combined with existing linear O-STBC decoders designed specifically for the time-selective channel, yielding significant performance improvement over conventionally constructed and decoded O-STBCs.**

***Index Terms*— Orthogonal space-time block codes, transmit diversity, time-selective fading channels, self-interference.**

I. Introduction

ORTHOGONAL Space-Time Block Codes (O-STBCs) [1], [2] were designed based on the mathematical frame work of [3] and proved to fully exploit the diversity of multiple input multiple output (MIMO) non-correlated *quasi-static* (block) fading channels, while admitting a low-complexity linear symbol-by-symbol decoding procedure with maximum-likelihood decoding performance [4]. In spite of their rate-deficiency, therefore, unability to achieve the capacity of the MIMO block-fading channel [5], [6], O-STBCs remain an attractive low-complexity technique for MIMO channels with time selectivity severer than that of a block-fading channel.

The most critical problem that arises when O-STBCs are subjected to channels with symbol-by-symbol time-selectivity (hereafter referred to as *time-selective* channels) is the appearance of self-interference in the conventional symbol-by-symbol decoder. This loss of orthogonality of channel matrix in the time-selective channel has been addressed by several authors.

In [7], [8], Kalman filter is introduced in the decoder in order to track the channel variation within each code block. Besides being limited to the Alamouti scheme, these decoders are designed to provide maximum diversity gain which, unfortunately, in the presence of time-selectivity, leads to error floors at high signal-to-noise ratios due to self-interference. In [9], an iterative self-interference cancelation technique for O-STBCs, capable of reducing the error-floor caused by the time-selective channel as the algorithm iterates, was proposed. Finally, elegant zero-forcing decoders that ensure orthogonal (symbol-by-symbol) decoding by removing the self-interference at the price of a loss in the transmit diversity gain achieved have been proposed in [10] and [11], [12], with the technique in [11], [12] being a systematic method to orthogonally decode O-STBCs of any number of transmit antennas in the time-selective fading channel.

All the aforementioned techniques share the fact that performance degradation of O-STBCs is dealt with at the receiver, therefore increasing its computational complexity. In contrast, following the same train of thought of [13], [14], which manipulates the original O-STBCs transmit scheme, a new transmit oriented method to combat this problem is proposed in this paper with the peculiarity of no need for matrix inversions at the decoder. The approach is complimentary to the receiver-oriented techniques previously described, and consists of a simple modification to the rate $\frac{1}{2}$ O-STBCs encoding matrices according to a self-interference minimization criterion.

The proposed modification of O-STBCs do not sacrifice generality, rate, diversity gain or the decoding complexity of conventional O-STBCs, leading to a family of *modified* O-STBCs (MO-STBCs), which benefit from increased robustness to time-selectivity in the channel. The construction of MO-STBCs from conventional O-STBCs is systematic, and can be applied to any number of transmit antennas and arbitrary complex constellations.

II. O-STBCs in the Time-selective Fading Channel

Consider an O-STBC system with n_t transmit antennas. Let $\mathbf{X}_{n_t}$ be the rate-one generalized real orthogonal design of size n_t as derived in [2]. The matrix $\mathbf{X}_{n_t}$ is $N \times n_t$, where N is the minimum delay which grows exponentially with n_t and is given by [15],

$$N = 16^{\lfloor (n_t-1)/8 \rfloor} 2^{\lceil \log_2(1+\mathrm{mod}(n_t-1,8)) \rceil}, \qquad (1)$$

where $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ are the flooring and ceiling functions, respectively.

Recall that the generalized real orthogonal design with rate-one ensures that there are N and only N distinct symbols x_i in $\mathbf{X}_{n_t}$. Therefore, $\mathbf{X}_{n_t}$ can be seen as the image of the real

Gabriel Porto Villardi and Prof. Ryuji Kohno are with the Yokohama National University, Dept. of Elec. & Comp. Eng. Yokohama-shi, Hodogaya-ku, Tokiwadai 79-5, Japan-240-8501. e-mail: [gpvillardi;kohno]@kohnolab.dnj.ynu.ac.jp

Prof. Giuseppe Abreu is with the University of Oulu, Centre for Wireless Communications, P.O. Box 4500, 90014-Finland. e-mail: giuseppe@ee.oulu.fi

vector $\mathbf{x} = [x_1\ x_2\ \cdots\ x_N]^{\mathrm{T}}$, where $^{\mathrm{T}}$ denotes transpose, under the single-valued function $\mathcal{X}_{n_t}$,

$$\mathbf{X}_{n_t} = \mathcal{X}_{n_t}(\mathbf{x}). \tag{2}$$

Over arbitrary complex constellations $\mathcal{S}$, the corresponding orthogonal space-time block encoding matrix assumes the following structure:

$$\mathcal{G}_{n_t} = \begin{cases} \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} & \text{if } n_t = 2, \\ \begin{bmatrix} \mathcal{X}_{n_t}(\mathbf{s}) \\ \hdashline \mathcal{X}_{n_t}(\mathbf{s})^* \end{bmatrix} & \text{if } n_t > 2, \end{cases} \tag{3}$$

where * denotes complex conjugate and $\mathbf{s} = [s_1\ s_2\ \cdots\ s_N]^{\mathrm{T}}$ is a vector of N distinct symbols taken from $\mathcal{S}$.

The received signal vector corresponding to the transmission of $\mathcal{G}_{n_t}$ through a time-selective MISO channel $\mathbf{H} \in \mathbb{C}^{2N \times n_t}$ [7]–[10], subject to zero-mean additive white Gaussian noise (AWGN) with variance $\sigma_w^2/2$ per dimension can be written as [4],

$$\begin{aligned} \mathbf{r} &= \mathrm{diag}(\mathcal{G}_{n_t} \cdot \mathbf{H}^{\mathrm{T}}) + \mathbf{w} \\ &= \Big[\ \mathcal{H}_N(\mathbf{H}_{\{1:N,:\}}) \ \Big|\ \mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})\Big] \cdot \begin{bmatrix} \mathbf{s} \\ \hdashline \mathbf{s}^* \end{bmatrix} + \mathbf{w}, \end{aligned} \tag{4}$$

where

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_t} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,n_t} \\ \hdashline h_{N+1,1} & \cdots & h_{N+1,n_t} \\ \vdots & \ddots & \vdots \\ h_{2N,1} & \cdots & h_{2N,n_t} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{\{1:N,:\}} \\ \hdashline \mathbf{H}_{\{N+1:2N,:\}} \end{bmatrix}, \tag{5}$$

$\mathbf{H}_{\{1:N,:\}}$ denotes (in Matlab style) the N-row partition of $\mathbf{H}$ and $\mathcal{H}_N$ is a function that [4],

$$\mathrm{diag}\big(\mathcal{X}_{n_t}(\mathbf{s}) \cdot \mathbf{H}^{\mathbf{T}}_{\{1:N,:\}}\big) = \mathcal{H}_N(\mathbf{H}_{\{1:N,:\}}) \cdot \mathbf{s}. \tag{6}$$

Observe that (6) suggests that the $N \times N$ matrix $\mathcal{H}_N(\mathbf{H}_{\{1:\mathbf{N},:\}})$ can be seen as an equivalent "encoded" channel matrix, associated to $\mathcal{X}_{n_t}(\mathbf{s})$.

Assuming that the receiver has full knowledge of the channel matrix $\mathbf{H}$, the conventional coherent matched-filter receiver of O-STBCs under the time-selective fading channel can be written, yielding [4], [9]–[12],

$$\begin{aligned} \hat{\mathbf{s}} &= \Big[\mathcal{H}_N(\mathbf{H}_{\{1:N,:\}})^{\mathrm{H}} \ \Big|\ \mathbf{0}\Big] \cdot \mathbf{r} + \Big[\mathbf{0} \ \Big|\ \mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})^{\mathrm{T}}\Big] \cdot \mathbf{r}^* \\ &= \bar{\mathbf{D}} \cdot \mathbf{s} + \bar{\mathbf{w}}, \end{aligned} \tag{7}$$

where $^{\mathrm{H}}$ denotes transpose conjugate (Hermitian), $\bar{\mathbf{w}}$ is a transformed noise vector with the same statistics of $\mathbf{w}$ (see [1], [16]) and

$$\begin{aligned} \bar{\mathbf{D}} &= \mathcal{H}_N(\mathbf{H}_{\{1:N,:\}})^{\mathrm{H}} \cdot \mathcal{H}_N(\mathbf{H}_{\{1:N,:\}}) \\ &\quad + \mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})^{\mathrm{T}} \cdot \mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})^* \\ &= [\bar{d}_{i,j}]_{N \times N}. \end{aligned} \tag{8}$$

Notice that since $\mathcal{H}_N(\cdot)$ is not an unitary matrix for $N > 2$ [2, Corollary 5.4.1], (8) has in general $\bar{d}_{i,j} \neq 0$ for $i \neq j$. Physically, this nondiagonality, leads to self-interference of which significance depends upon the time-selectivity of the channel.

III. Systematic MO-STBCs design

In this section we introduce a simple technique which reduces the self-interference present in the estimation of a given symbol by modifying the rate $\frac{1}{2}$ transmit scheme introduced in [2], but preserving the same code structure and complexity.

We define the MO-STBCs over complex constellations, as:

$$\check{\mathcal{G}}_{n_t} = \begin{cases} \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} & \text{if } n_t = 2, \\ \begin{bmatrix} \check{\mathcal{X}}_{n_t}(\mathbf{s}) \\ \hdashline \check{\mathcal{X}}_{n_t}(\mathbf{s})^* \end{bmatrix} & \text{if } n_t > 2, \end{cases} \tag{9}$$

where $\check{\mathcal{X}}_{n_t}$ is the function $\mathcal{X}_{n_t}$ modified to perform conjugate operations in an "*every-other-line*" fashion,

$$\check{\mathcal{X}}_{n_t}(\mathbf{s})_{\{1:2:N,:\}} = \mathcal{X}_{n_t}(\mathbf{s})_{\{1:2:N,:\}}, \tag{10}$$

$$\check{\mathcal{X}}_{n_t}(\mathbf{s})_{\{2:2:N,:\}} = \mathcal{X}_{n_t}(\mathbf{s})^*_{\{2:2:N,:\}}. \tag{11}$$

For two transmit antennas, MO-STBC is identical to the Alamouti scheme [1]. For a larger number of transmit antennas MO-STBCs preserve the same structure and properties of O-STBCs, however, its conjugate operations are carefully spread through the entire encoding matrix, rather than being concentrated on the lower half of the matrix, as in [2].

As with conventional O-STBCs, the received signal vector corresponding to the transmission of the code $\check{\mathcal{G}}_{n_t}$ through the channel $\mathbf{H}$, subject to noise can be written as [4],

$$\mathbf{r} = \mathrm{diag}(\check{\mathcal{G}}_{n_t} \cdot \mathbf{H}^{\mathrm{T}}) + \mathbf{w}, \tag{12}$$

where $\mathbf{H}$ is given by (5).

The coherent matched-filter receiver of MO-STBCs can be written as,

$$\begin{aligned} \hat{\mathbf{s}} &= \Big[\check{\mathcal{H}}_N(\mathbf{H}_{\{1:N,:\}})^{\mathrm{H}} \ \Big|\ \mathbf{0}\Big] \cdot \check{\mathbf{r}} + \Big[\mathbf{0} \ \Big|\ \check{\mathcal{H}}_N(\mathbf{H}_{\{N+1:2N,:\}})^{\mathrm{T}}\Big] \cdot \check{\mathbf{r}}^* \\ &= \check{\mathbf{D}} \cdot \mathbf{s} + \check{\mathbf{w}}, \end{aligned} \tag{13}$$

where,

$$\begin{aligned} \check{\mathbf{D}} &= \check{\mathcal{H}}_N(\mathbf{H}_{\{1:N,:\}})^{\mathrm{H}} \cdot \check{\mathcal{H}}_N(\mathbf{H}_{\{1:N,:\}}) \\ &\quad + \check{\mathcal{H}}_N(\mathbf{H}_{\{N+1:2N,:\}})^{\mathrm{T}} \cdot \check{\mathcal{H}}_N(\mathbf{H}_{\{N+1:2N,:\}})^* \\ &= [\check{d}_{i,j}]_{N \times N}, \end{aligned} \tag{14}$$

with

$$\check{\mathcal{H}}_N(\mathbf{H}_{\{1:2:N,:\}}) = \mathcal{H}_N(\mathbf{H}_{\{1:2:N,:\}}), \tag{15}$$

$$\check{\mathcal{H}}_N(\mathbf{H}_{\{2:2:N,:\}}) = \mathcal{H}_N(\mathbf{H}_{\{2:2:N,:\}})^*. \tag{16}$$

The received vector $\mathbf{r}$ must be put in the same conjugation operation fashion, thus

$$\check{\mathbf{r}}_{\{1:2:2N\}} = \mathbf{r}_{\{1:2:2N\}}, \tag{17}$$

$$\check{\mathbf{r}}_{\{2:2:2N\}} = \mathbf{r}^*_{\{2:2:2N\}}. \tag{18}$$

The construction of MO-STBCs can be better understood via a concrete example. Consider a four transmit - one receive antennas O-STBC system. The code $\mathcal{G}_4$, $\mathcal{H}_4(\mathbf{H}_{\{\mathbf{1:4},:\}})$, $\mathcal{H}_4(\mathbf{H}_{\{\mathbf{5:8},:\}})$ and (12) are modified into

$$\check{\mathcal{H}}_4(\mathbf{H}_{\{\mathbf{1:4},:\}}) = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,2}^* & -h_{2,1}^* & h_{2,4}^* & -h_{2,3}^* \\ h_{3,3} & -h_{3,4} & -h_{3,1} & h_{3,2} \\ h_{4,4}^* & h_{4,3}^* & -h_{4,2}^* & -h_{4,1}^* \end{bmatrix},$$

$$\check{\mathcal{H}}_4(\mathbf{H}_{\{\mathbf{5:8},:\}}) = \begin{bmatrix} h_{5,1} & h_{5,2} & h_{5,3} & h_{5,4} \\ h_{6,2}^* & -h_{6,1}^* & h_{6,4}^* & -h_{6,3}^* \\ h_{7,3} & -h_{7,4} & -h_{7,1} & h_{7,2} \\ h_{8,4}^* & h_{8,3}^* & -h_{8,2}^* & -h_{8,1}^* \end{bmatrix},$$

$$\check{\mathcal{G}}_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4^* & -s_3^* & s_2^* & s_1^* \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4 & -s_3 & s_2 & s_1, \end{bmatrix},$$

$$\check{\mathbf{r}} = \begin{bmatrix} r_1 & r_2^* & r_3 & r_4^* & r_5 & r_6^* & r_7 & r_8^* \end{bmatrix}^{\mathrm{T}}. \quad (19)$$

As for the complexity of the MO-STBCs decoder is concerned, it should be noticed that if the K different symbols in each transmitted MO-STBC block belong to a constellation A with Q elements, the block is decoded by computing (13) and, then, searching A for the symbols that holds the shortest distance to each element of (13). This process involves computing the metric M, Q times, for each of the k^{th} element ,

$$M_{(k,q)} = \left|\hat{s}_k - s_q\right|_{q=1,\ldots,Q}. \quad (20)$$

Therefore, the complexity involved in decoding one MO-STBC block is $\mathcal{O}(QK)$, which is the same as the one required for decoding one O-STBC block.

IV. Self-Interference Analysis

A simultaneously explicit and general formula for the off-diagonal elements of (8) and (14) cannot be written, since the transformation $\mathcal{H}_N$ is code-dependent. We shall, therefore, resort to a statistic characterization of the self-interference caused by the non-diagonality of (8) and (14).

The severity of the time-selectivity present in (5) is inversely proportional the temporal autocorrelation of its branches (columns). The autocorrelation coefficient of independent and identically distributed (i.i.d.) fading channels with Gaussian independent real and imaginary components has the closed-form [17], [18],

$$\alpha_\Delta = \frac{E[h_{i,n}h_{i+\Delta,n}^*]}{\sigma_{h_n}^2} = J_0(2\pi f_D T\Delta), \quad (21)$$

where $E[\cdot]$ denotes expectation, $\sigma_{h_n}^2$ is the variance of the n-th channel branch h_n, J_0 is the Bessel function of the first kind and zero-th order, f_D is the maximum Doppler frequency of the channel, T is the time period between successive transmissions and Δ is the difference between the indexes of two transmit epochs.

At one extreme is the case when $\alpha_\Delta = 1$ (block-fading channel), where it is known that $\mathcal{H}_N(\mathbf{H}_{\{1:N,:\}})$ and $\mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})$ are not unitary [2, Corollary 5.4.1]. At the other, is the case when $\alpha_\Delta \to 0$ (fast-fading channel), where $\mathcal{H}_N(\mathbf{H}_{\{1:N,:\}})$ and $\mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})$ reduce to complex random matrices approaching unitary matrices as N increases by force of the Law of Large Numbers. Most cases of interest are, however, those with not too many transmit antennas and moderate time-selectivity, where the relative diversity advantage obtained with O-STBCs is most significant [9]–[12] and the channel can be effectively estimated [7], [8], [19]. In such cases, $\mathcal{H}_N(\mathbf{H}_{\{1:N,:\}})$ and $\mathcal{H}_N(\mathbf{H}_{\{N+1:2N,:\}})$ are almost surely not simultaneously unitary and, therefore, (8) and (14) are almost surely not diagonal.

A. For O-STBCs

Consider, for instance, the code $\mathcal{G}_4$ in [2]. The off-diagonal elements $\bar{d}_{1,2}$, $\bar{d}_{1,3}$ and $\bar{d}_{1,4}$ of (8), which, represent the interference caused by symbols s_2, s_3 and s_4 onto the estimate of s_1, are

$$\bar{d}_{1,2} = h_{1,2}h_{1,1}^* - h_{2,2}^*h_{2,1} - h_{3,4}h_{3,3}^* + h_{4,4}^*h_{4,3} + h_{5,2}^*h_{5,1} - h_{6,2}h_{6,1}^* - h_{7,4}^*h_{7,3} + h_{8,4}h_{8,3}^*, \quad (22)$$

$$\bar{d}_{1,3} = h_{1,3}h_{1,1}^* + h_{2,2}^*h_{2,4} - h_{3,1}h_{3,3}^* - h_{4,4}^*h_{4,2} + h_{5,3}^*h_{5,1} + h_{6,2}h_{6,4}^* - h_{7,1}^*h_{7,3} - h_{8,4}h_{8,2}^*, \quad (23)$$

$$\bar{d}_{1,4} = h_{1,4}h_{1,1}^* - h_{2,2}^*h_{2,3} + h_{3,2}h_{3,3}^* - h_{4,4}^*h_{4,1} + h_{5,4}^*h_{5,1} - h_{6,2}h_{6,3}^* + h_{7,2}^*h_{7,3} - h_{8,4}h_{8,1}^*. \quad (24)$$

Next, consider $\sigma_{\bar{d}_{1,2}}^2$, which can be calculated as follows. First, split (22) into the terms $p_1 = (h_{1,2}h_{1,1}^* - h_{6,2}h_{6,1}^*)$; $p_2 = (-h_{3,4}h_{3,3}^* + h_{8,4}h_{8,3}^*)$; $p_3 = (h_{5,2}^*h_{5,1} - h_{2,2}^*h_{2,1})$ and $p_4 = (-h_{7,4}^*h_{7,3} + h_{4,4}^*h_{4,3})$. Then, under the assumption of equipower diversity branches with a unitary sum ($\sigma_{h_1}^2 = \sigma_{h_2}^2 = \sigma_{h_3}^2 = \sigma_{h_4}^2 = \frac{1}{4}$), $\sigma_{p_1}^2$ becomes

$$\begin{aligned} E[p_1p_1^*] &= E[(h_{1,2}h_{1,1}^* - h_{6,2}h_{6,1}^*)(h_{1,2}^*h_{1,1} - h_{6,2}^*h_{6,1})] \\ &= \sigma_{h_{1,2}}^2\sigma_{h_{1,1}}^2 + \sigma_{h_{6,2}}^2\sigma_{h_{6,1}}^2 - E[h_{1,2}h_{1,1}^*h_{6,2}^*h_{6,1}] \\ &\quad - E[h_{6,2}h_{6,1}^*h_{1,2}^*h_{1,1}] \\ &= 2(\sigma_{h_2}^2\sigma_{h_1}^2) - 2\big(J_0^2(2\pi f_D T5)\sigma_{h_2}^2\sigma_{h_1}^2\big) \\ &= \frac{(1-\alpha_5^2)}{8}. \end{aligned} \quad (25)$$

Repeating this calculation for the remaining terms p_2, p_3 and p_4, and adding them up, the variance of the instantaneous self-interference originated by symbol s_2 is found to be

$$\sigma_{\bar{d}_{1,2}}^2 = \frac{(1-\alpha_3^2)}{4} + \frac{(1-\alpha_5^2)}{4}. \quad (26)$$

The variances $\sigma_{\bar{d}_{1,3}}^2$ and $\sigma_{\bar{d}_{1,4}}^2$, which are, respectively, the variance of the instantaneous self-interference caused by symbols s_3 and s_4 onto s_1, can be calculated likewise, yielding

$$\sigma_{\bar{d}_{1,3}}^2 = \frac{(1-\alpha_2^2)}{4} + \frac{(1-\alpha_6^2)}{4}, \quad (27)$$

$$\sigma_{\bar{d}_{1,4}}^2 = \frac{(1-\alpha_1^2)}{8} + \frac{(1-\alpha_3^2)}{8} + \frac{(1-\alpha_5^2)}{8} + \frac{(1-\alpha_7^2)}{8}. \quad (28)$$

Adding (26)-(28), we obtain

$$\Upsilon = \frac{(1-\alpha_1^2)}{8}+\frac{(1-\alpha_2^2)}{4}+\frac{3(1-\alpha_3^2)}{8} + \frac{3(1-\alpha_5^2)}{8}+\frac{(1-\alpha_6^2)}{4}+\frac{(1-\alpha_7^2)}{8}, \quad (29)$$

which is the variance of the total self-interference affecting the estimation of symbol s_1, and can be also put on the form of

$$\Upsilon = \frac{\big(1-J_0^2(2\pi f_D T)\big)}{8}+\frac{\big(1-J_0^2(4\pi f_D T)\big)}{4} +\frac{3\big(1-J_0^2(6\pi f_D T)\big)}{8}+\frac{3\big(1-J_0^2(10\pi f_D T)\big)}{8} +\frac{\big(1-J_0^2(12\pi f_D T)\big)}{4}+\frac{\big(1-J_0^2(14\pi f_D T)\big)}{8}, \quad (30)$$

to evidence the self-interference's dependence on the channel fading rate. From (29) and (30) one can readily deduce that an important strategy to combat the effects caused by the time-selective channel is to make the autocorrelation coefficients approach unity $\big(\alpha = J_0(\cdot) \rightarrow 1\big)$, that is to say, to minimize Δ of (21).

B. For MO-STBCs

In this subsection it will be shown that a minimization of the self-interference, which impairs O-STBCs orthogonality, is indeed obtained by the modification to the O-STBCs encoding matrices presented in section III, rather than being restricted to special types of decoders [7]–[12].

Consider MO-STBC's $\check{\mathcal{G}}_4$, corresponding to $\mathcal{G}_4$. The self-interference analysis for MO-STBCs is performed in the same way as for conventional O-STBCs. We, first, quantify the off-diagonal elements $\check{d}_{1,2}$, $\check{d}_{1,3}$, and $\check{d}_{1,4}$ of (14), which are, respectively, the instantaneous self-interference that symbols s_2, s_3 and s_4, cause onto the estimation of s_1. We, then, have

$$\check{d}_{1,2} = h_{1,2}h_{1,1}^* - h_{2,2}h_{2,1}^* - h_{3,4}h_{3,3}^* + h_{4,4}h_{4,3}^* + h_{5,2}^*h_{5,1} - h_{6,2}^*h_{6,1} - h_{7,4}^*h_{7,3} + h_{8,4}^*h_{8,3}, \quad (31)$$

$$\check{d}_{1,3} = h_{1,3}h_{1,1}^* + h_{2,2}h_{2,4}^* - h_{3,1}h_{3,3}^* - h_{4,4}h_{4,2}^* + h_{5,3}^*h_{5,1} + h_{6,2}^*h_{6,4} - h_{7,1}^*h_{7,3} - h_{8,4}^*h_{8,2}, \quad (32)$$

$$\check{d}_{1,4} = h_{1,4}h_{1,1}^* - h_{2,2}h_{2,3}^* + h_{3,2}h_{3,3}^* - h_{4,4}h_{4,1}^* + h_{5,4}^*h_{5,1} - h_{6,2}^*h_{6,3} + h_{7,2}^*h_{7,3} - h_{8,4}^*h_{8,1}. \quad (33)$$

Obviously (31)-(33) also do not reduce to zero, however, (31) and (33) yield a much smaller self-interference than their counterparts (22) and (24). This happens owing to the fact that through the application of the proposed scheme the product of channel samples $(h_{i,n}h_{i,m}^* - h_{i+\Delta,n}h_{i+\Delta,m}^*)$, that would cancel out in the block fading channel, are separated by a smaller Δ. Although (32) does not provide any reduction of self-interference compared to (23), the overall self-interference inherent to MO-STBCs in the time-selective channel is significantly smaller than the one of conventional O-STBCs. Now, by repeating the same analysis of subsection IV-A, we statistically characterize the self-interference caused by the non-diagonality of (14),

$$\sigma_{\check{d}_{1,2}}^2 = \frac{(1-\alpha_1^2)}{2}, \quad (34)$$

$$\sigma_{\check{d}_{1,3}}^2 = \frac{(1-\alpha_2^2)}{4}+\frac{(1-\alpha_6^2)}{4}, \quad (35)$$

$$\sigma_{\check{d}_{1,4}}^2 = \frac{(1-\alpha_1^2)}{4}+\frac{(1-\alpha_3^2)}{4}. \quad (36)$$

The total self-interference is significantly reduced and given by adding (34)-(36),

$$\check{\Upsilon} = \frac{3(1-\alpha_1^2)}{4}+\frac{(1-\alpha_2^2)}{4}+\frac{(1-\alpha_3^2)}{4}+\frac{(1-\alpha_6^2)}{4}, \quad (37)$$

which can be put on the form of

$$\check{\Upsilon} = \frac{3\big(1-J_0^2(2\pi f_D T)\big)}{4}+\frac{\big(1-J_0^2(4\pi f_D T)\big)}{4} +\frac{\big(1-J_0^2(6\pi f_D T)\big)}{4}+\frac{\big(1-J_0^2(12\pi f_D T)\big)}{4}. \quad (38)$$

Now, if we subtract (38) from (30), we have

$$\Upsilon - \hat{\Upsilon} = \frac{-5\big(1-J_0^2(2\pi f_D T)\big)}{8}+\frac{\big(1-J_0^2(6\pi f_D T)\big)}{8} +\frac{3\big(1-J_0^2(10\pi f_D T)\big)}{8}+\frac{\big(1-J_0^2(14\pi f_D T)\big)}{8}, \quad (39)$$

which can be analyzed for $0 \leqslant f_D T \leqslant X$, where we define X to be the $f_D T$ value that makes the channel autocorrelation coefficient through a transmitted block drops to zero. That is to say, the first and last symbol of the same block are totally uncorrelated. For the code $\mathcal{G}_4$, $\alpha_7 = 0$ leads to $X \approx 0.054$. Letting $f_D T = 0$, $f_D T = X$ and $0 < f_D T < X$, respectively, yields

$$\Upsilon - \hat{\Upsilon} = \frac{-5\big(1-J_0^2(0)\big)}{8}+\frac{\big(1-J_0^2(0)\big)}{8}+\frac{3\big(1-J_0^2(0)\big)}{8} +\frac{\big(1-J_0^2(0)\big)}{8} = 0, \quad (40)$$

$$\Upsilon - \hat{\Upsilon} = \frac{-5\big(1-J_0^2(X)\big)}{8}+\frac{\big(1-J_0^2(X)\big)}{8}+\frac{3\big(1-J_0^2(X)\big)}{8} +\frac{\big(1-J_0^2(X)\big)}{8} \approx 0.46, \quad (41)$$

$$\Upsilon - \hat{\Upsilon} = \frac{-5\big(1-J_0^2(2\pi f_D T)\big)}{8}+\frac{\big(1-J_0^2(6\pi f_D T)\big)}{8} + \frac{3\big(1-J_0^2(10\pi f_D T)\big)}{8}+\frac{\big(1-J_0^2(14\pi f_D T)\big)}{8} > 0, \quad (42)$$

since $\big(1-J_0^2(14\pi f_D T)\big) > \big(1-J_0^2(10\pi f_D T)\big) > \big(1-J_0^2(6\pi f_D T)\big) > \big(1-J_0^2(2\pi f_D T)\big)$.

The analysis above show us that MO-STBCs benefits from smaller self-interference than of O-STBCs until high fading rates ($f_D T \approx 0.054$) and is graphically shown in Fig. 1. Moreover, the same is also true for extremely high values of $f_D T$ that already fall out of practical commercial applications, according to other analyses not included in this manuscript.

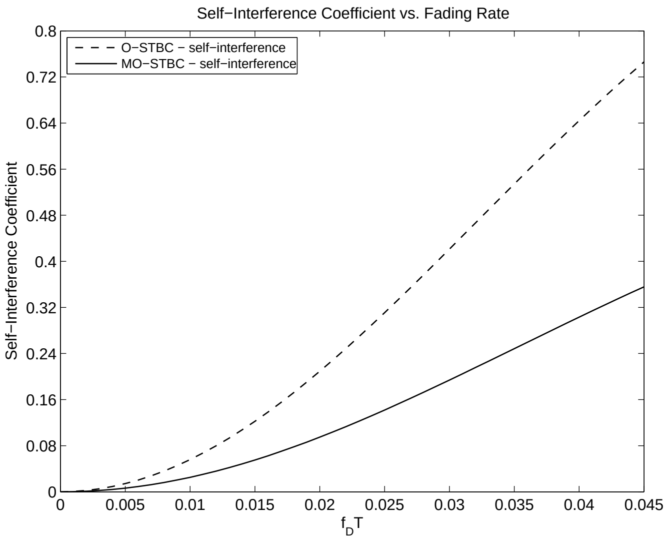

Fig. 1. Self-interference coefficient for different fading rates of a MO-STBC and O-STBC four transmit - one receive antennas system.

V. Theoretical Bit Error Rates under Generalized Fading Channels

Theoretical bit error rate expressions for O-STBCs under block fading with perfect CSI at the decoder, obtained in [20], [21], are extended to MO-STBCs with the decoder [4] under generalized time-selective fading channels. The method can be used to find the theoretical BER of coherent systems with any number of transmit antennas, however, the four transmit - one receive antennas system considered throughout this paper, with Gray code mapped M-ary PSK and QAM modulations will be evaluated to keep consistency with the analysis from section IV.

The bit error probabilities of rate ρ O-STBC code, as given by [20], [21], are

$$\bar{P}_{b:\text{PSK}}(\gamma|M) = \frac{1}{2\log_2 M}\sum_{k=1}^{M-1}\bar{d}_k\cdot\Big(I\big(\delta_k^-, g_{\text{PSK}}(\delta_k^-),4\big) - I\big(\delta_k^+, g_{\text{PSK}}(\delta_k^+),4\big)\Big), \quad (43)$$

$$\bar{P}_{b:\text{QAM}}(\gamma|M) = \frac{2}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\left[\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} d_i\cdot I\left(\frac{1}{2}, g_{\text{QAM}}(i),4\right)\right], \quad (44)$$

where, due to time-selectivity, $I(\cdot)$ becomes

$$I(\delta,g) = \frac{1}{\pi}\cdot\int_0^{\pi(1-\delta)}\prod_{n=1}^{4}\mu_{\gamma_n}\left(-\frac{g}{\rho\cdot\sin^2(\theta)}\right)d\theta, \quad (45)$$

$$g_{\text{PSK}}(\delta) = \sin^2(\pi\delta), \quad (46)$$

$$\delta_k^- = \frac{2k-1}{M}, \quad (47)$$

$$\delta_k^+ = \frac{2k+1}{M}, \quad (48)$$

$$\bar{d}_{k:\text{PSK}} = 2\left|\frac{k}{M}+\left\lfloor\frac{k}{M}\right\rceil\right| + 2\sum_{i=2}^{\log_2 M>2}\left|\frac{k}{2^i}+\left\lfloor\frac{k}{2^i}\right\rceil\right|, \quad (49)$$

for uniform PSK constellations, in which M represents the adopted constellation size, $|\cdot|$ denotes absolute value and the function $\lfloor x\rceil$ rounds x to the closest integer. For QAM modulation,

$$g_{\text{QAM}}(i) = \frac{3(2i+1)^2}{2(M-1)}, \quad (50)$$

$$d_i = (-1)^{\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}\right\rfloor}\left(2^{k-1}-\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right). \quad (51)$$

Following, the MGF associated with the non-correlated and balanced paths, is given by

$$\mu_{\gamma_n}(s) = \left(1-s\frac{1}{4}\gamma_n\right)^{-1}, \quad (52)$$

where γ_n, now, is the Signal-to-Interference-plus-Noise Ratio (SINR), owing to the self-interference (38). The inclusion of the denominator 4 results from the power normalization at the transmission.

All the procedure described above can be easily extended from Rayleigh to Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels by substituting its MGF, respectively, by

$$\mu_{\gamma_n}(s) = \left(1-s\frac{\gamma_n}{2}+\frac{(s\frac{\gamma_n}{2})^2 q^2}{(1+q^2)^2}\right)^{-1/2}, \quad (53)$$

$$\mu_{\gamma_n}(s) = \frac{(1+n^2)}{(1+n^2)-s\frac{\gamma_n}{4}}\exp\left(\frac{n^2 s\frac{\gamma_n}{4}}{(1+n^2)-s\frac{\gamma_n}{4}}\right), \quad (54)$$

where q $(0\leqslant q\leqslant 1)$ and n $(0\leqslant n)$ are the fading parameters and their one-to-one mapping with the m parameter is described in [22].

The difference between MO-STBCs and O-STBCs with the receiver [4] is only the self-interference coefficient factor, given by (38) and (30). Thus, the method described in this section can be used straightforwardly to calculate the theoretical bit error probabilities of O-STBCs.

VI. Simulation and Results

In this paper O-STBCs and MO-STBCs are analyzed under frequency flat time-selective fading channels as in [7]–[10]. However, rather than adopting a simple AR(1) model, as in the aforementioned references, all simulations in this paper followed the Jakes model to generate channels with variable fading rate $f_D T$, unless stated otherwise. As described in [10] (and references therein), the assumption of a channel invariant over consecutive symbols is not always realistic. Therefore, the time selectivity here considered is of a channel whose value is constant during one symbol transmission interval, but slightly changes between adjacent symbols. As in [7]–[12], [19], no correlation in the space domain was considered. Moreover, 8-PSK and 16-QAM modulations are employed and the channel is estimated based on the Adaptive Frame-based Interpolation method proposed by [19].

Fig. 2 shows the BER performance of 8-PSK modulated four and eight transmit - one receive antennas systems with

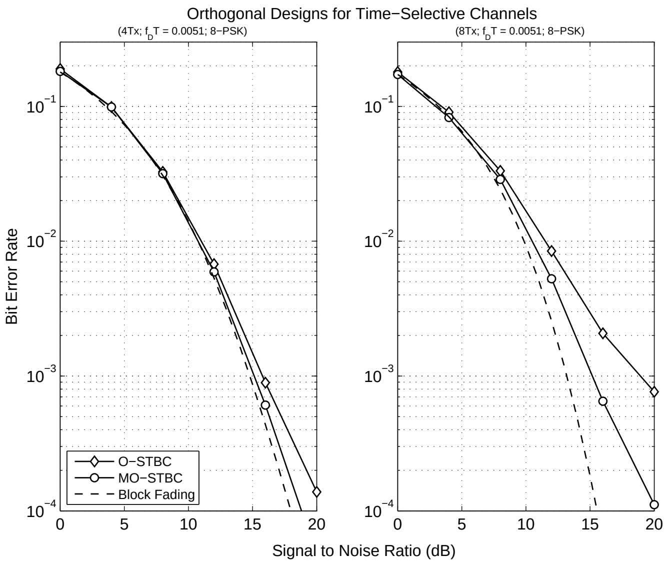

Fig. 2. BER for four and eight transmit - one receive antennas systems employing MO-STBC and O-STBC as the encoding scheme under a fading rate of $f_D T$ = 0.0051 (8-PSK modulation).

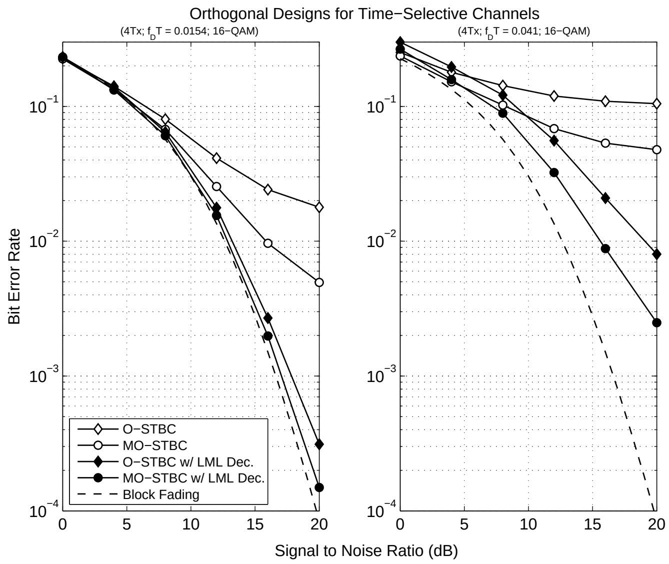

Fig. 3. BER for a four transmit - one receive antennas systems employing MO-STBC and O-STBC as the encoding scheme and their combination with the Linear Maximum Likelihood Decoder [11], [12] under fading rates of $f_D T$ = 0.0154 and $f_D T$ = 0.041 (16-QAM modulation).

$f_D T = 0.0051$. It can be noticed that MO-STBC mitigates the loss of orthogonality in symbol decoding (minimizing the self-interference), thus considerably outperforms conventional O-STBC in the presence of time-selective fading channels. However, for scenarios where the self-interference is naturally bigger, such as systems employing a large number of transmit antennas or when higher fading rates are experienced, performance degradation is significant regardless the adopted scheme. This suggests the need of combining MO-STBCs with a time-selectivity robust decoding technique for such high self-interference scenarios.

Fig. 3 shows a 16-QAM modulated system with four transmit - one receive antennas, experiencing higher fading rates ($f_D T = 0.0154$ and $f_D T = 0.041$), which can be translated as larger symbol-to-symbol channel variations. It can be seen that when MO-STBC is combined with the receive robust technique [11], [12], the diversity loss inherent to this decoder is minimized to a level where the performance degradation is less than 0.5dB for a bit error rate of 10^{-3} with $f_D T = 0.0154$. For the previously considered bit error rate and fading rate, the aforementioned combination yields about 1dB improvement compared to O-STBC combined with the robust decoder. When the fading rate is further increased to $f_D T = 0.041$, the combination of MO-STBC with the robust decoder yields almost 4dB improvement for a 10^{-2} bit error rate.

Fig. 4 shows the performance of a 16-QAM modulated four transmit - one receive antennas system with different configurations under various fading rates with $SNR = 20dB$. In agreement with the analysis presented in section IV, MO-STBCs also achieve the ideal block fading performance when no time-selectivity is present. It can be observed that for all fading rates considered MO-STBC significantly outperforms O-STBC scheme.

The accuracy of the method developed in section V to calculate theoretical bit error probabilities can be confirmed by the comparison with simulated values shown in Fig. 5. It can be seen that MO-STBCs and O-STBCs are precisely computed for different fading rates, regardless the channel statistics. This allows for quick evaluation of the considered systems in many different scenarios, thus cumbersome and time-consuming computer simulations can be avoided.

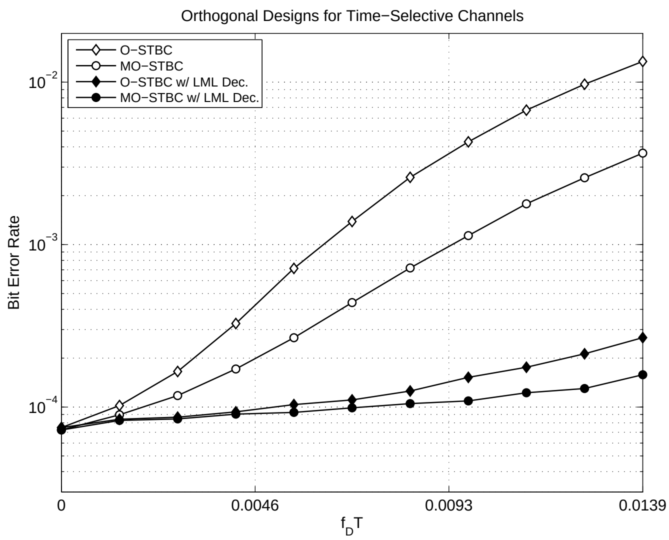

Fig. 4. BER for a four transmit - one receive antennas system with different $f_D T$ values and 16-QAM Modulation, employing MO-STBC and O-STBC as the encoding scheme and their combination with the Linear Maximum Likelihood Decoder [11], [12] (SNR = 20dB).

VII. Conclusions

We have proposed a new scheme, named MO-STBC, which improves significantly the robustness against time-selectivity though having exactly the same complexity as the conventional

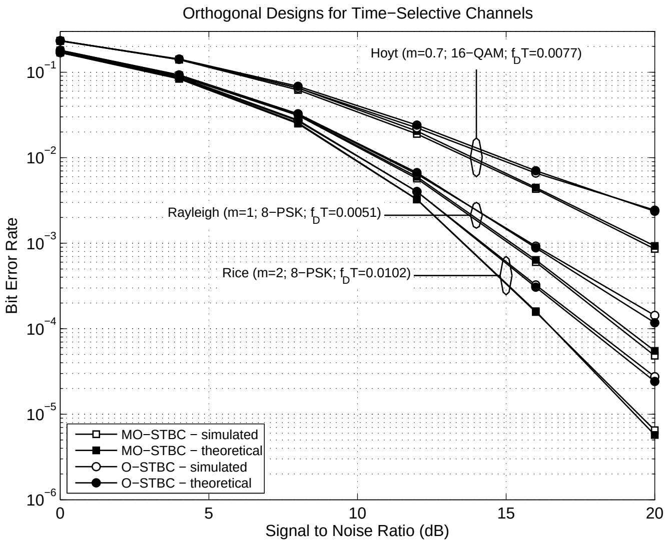

Fig. 5. Theoretical and simulated BER for a four transmit - one receive antennas system employing MO-STBC and O-STBC as the encoding scheme under Hoyt (m=0.7; 16-QAM; $f_D T$ = 0.0077), Rayleigh (m=1; 8-PSK; $f_D T$ = 0.0051) and Rice (m=2; 8-PSK; $f_D T$ = 0.0102) fading channels.

O-STBC. This improvement, which is achieved by a non-trivial, yet simple modification to the encoding scheme of O-STBC, makes MO-STBC a significant ally to the already developed time-selective robust decoding techniques. Very accurate theoretical bit error probabilities for MO-STBCs and O-STBCs over generalized time-selective fading channels, with Gray code mapped M-ary PSK and QAM modulations, were also derived.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 11, no. 8, pp. 1451 – 1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456 – 1467, July 1999.

[3] A. V. Geramita and J. Seberry, *Orthogonal Designs: Quadractic Forms and Hadamard Matrices*. New York-Basel: Marcel-Dekker, 1979.

[4] X. Li, T. Luo, G. Yue, and C. Yin, "A squaring method to simplify the decoding of orthogonal space-time block codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1700 – 1703, Oct. 2001.

[5] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585 – 595, Nov./Dec. 1999.

[6] S. Sandhu and A. Paulraj, "Space-time block codes: a capacity perspective," *IEEE Commun. Lett.*, vol. 4, no. 12, pp. 384 – 386, Dec. 2000.

[7] Z. Liu, X. Ma, and G. B. Giannakis, "Space-time coding and Kalman filtering for time-selective fading channels," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 183 – 186, Feb. 2002.

[8] S. O. K. Teo and T. Hinamoto, "Kalman filter based channel estimation for space-time code," in *Proc. IEEE The* 47^{th} *International Midwest Symposium on Circuits and Systems*, vol. 2, July, 25 - 28 2004, pp. 669 – 672.

[9] F. C. Zheng and A. G. Burr, "Signal detection of orthogonal space-time block coding over time-selective fading channels: a pic approach for the $\}_i$ systems," *IEEE Trans. Commun.*, vol. 53, no. 6, pp. 969 – 972, 2005.

[10] T. A. Tran and A. B. Sesay, "A generalized linear quasi-ML decoder of OSTBCs for wireless communications over time-selective fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 3, pp. 855 – 864, May 2004.

[11] G. T. F. de Abreu and R. Kohno, "Orthogonal decoding of space-time block codes in fast fading," in *Proc. IEEE International Symposium on Information Theory (ISIT'03)*, Yokohama, Japan, June 29 - July 4 2003, p. 155.

[12] G. T. F. de Abreu, H. Ochiai, and R. Kohno, "Linear maximum likelihood decoding of space-time block coded OFDM systems for mobile communications," *IEE Proceedings on Communications - Special Issue on Wireless LAN Systems and Internetworking*, vol. 151, no. 5, pp. 447 – 459, Oct. 2004.

[13] F. C. Zheng and A. G. Burr, "Receiver design for orthogonal space-time block coding for four transmit antennas over time-selective fading channels," in *Proc. IEEE Global Telecommunications Conference*, vol. 1, San Francisco, U.S.A., December 2003, pp. pages 128–132.

[14] ——, "Double diagonalisation: an improved zero-forcing detector for orthogonal stbc over time-selective fading channels," in *Proc. IEEE* 62^{nd} *VTC*, vol. 2, Dallas, U.S.A., September 2005, pp. pages 1060–1064.

[15] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. Inform. Theory*, vol. 48, no. 2, pp. 384 – 395, Feb. 2002.

[16] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Select. Areas Commun.*, vol. 17, no. 2, pp. 451 – 460, Mar. 1999.

[17] W. C. Jakes, *Microwave Mobile Communications*. New York, NY: Wiley, 1974.

[18] M. Pätzhold, U. Killat, F. Laue, and Y. Li, "On the statistical properties of deterministic simulation models for mobile fading channels," *IEEE Trans. Veh. Technol.*, vol. 47, no. 1, pp. 254 – 269, Feb. 1998.

[19] G. P. Villardi, G. T. F. de Abreu, and R. Kohno, "An adaptive frame-based interpolation method of channel estimation for space-time block codes in moderate fading channels," *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences - Special Issue on Multi-dimensional Signal Processing*, vol. E89-A, no. 3, pp. 660 – 669, 2006.

[20] G. T. F. de Abreu, "GABBA codes: Generalized full-rate orthogonally decodable space-time block codes," in *Proc. IEEE The* 39^th *Asilomar Conference on Signal, Systems and Computers (ASILOMAR'05)*, Monterey, USA, Oct 28 - Nov 1 2005.

[21] ——, "Generalized ABBA space-time block codes," *submited to Trans. IEEE Inform. Theory, under review*. [Online]. Available: http://arxiv.org/find/grp_cs/1/au:+Abreu/0/1/0/all/0/1

[22] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York, NY: Wiley, 2000.