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LOCALIZATION FROM IMPERFECT AND INCOMPLETE RANGING

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ABSTRACT

Source localization from imperfect and incomplete range information is considered. The problem is formulated as a combination of the well-known Euclidean Distance Matrix (EDM) approximation/completion problem and multidimensional scaling (MDS). A powerful technique that solves the EDM approximation/completion problem by exploiting the semi-definiteness property of a corresponding Euclidean kernel has been recently proposed [1]. That technique requires, however, that the entries of the input EDM be weighted in accordance to their reliability. In this paper, a formula for such a weight function, based on confidence-bound statistics of the distance estimates and on Graph spectral properties is studied. Computer simulations show that significant improvement in localization accuracy can be achieved by utilizing the weight function derived.

I. INTRODUCTION

Sensor localization from incomplete and imperfect range information has recently become an attractive research topic in the area of wireless communications. The analogous problem of estimating the relative location of multiple points from the corresponding set of dissimilarities has, however, been previously considered in areas such as psychology, geography and biology, and is known to be efficiently solved by multidimensional scaling (MDS) [2].

When the dissimilarity between points is the Euclidean distance, the matrices of all pairwise metrics is known as the *Euclidean Distance Matrix* (EDM) and the corresponding MDS technique is referred to as *metric* MDS. The parallel between metric MDS and the sensor localization problem has been recently considered in [3–5].

In comparison to "classic" MDS [2], however, the new problem brings new challenges. In particular, in typical wireless network localization problems, ranging information, collected by non-identical devices, is often imperfect, incomplete and follow different statistics. Therefore, robust and accurate non-parametric algorithms that handle incomplete and imperfect distance estimates are desired.

In this paper, one such algorithm is proposed. First, the sensor localization problem is formulated in terms of the EDM approximation/completion problem posed and solved in [1]. The technique, which requires that the entries of the input EDM be weighted according to their reliability, is then brought into the context of sensor localization by the derivation of a weight-function computed from the distance estimates and the structural property of the network.

The remainder of the paper is structured as follows.

In section II, the problem and assumptions are stated formally and put into context. That section also serves the purpose of establishing the terminology and mathematical notation used throughout the paper. In section III, the application of MDS to sensor localization is briefly reviewed and the semi-definite programming (SDP) solution for EDM completion/approximation problems (EDM-CP/AP) is discussed. In section IV, the proposed weight function is derived and the performance of its application to the SDP-based localization technique [1], henceforth referred to as *modified* SDP, is studied in section V. A brief conclusion is given in section VI.

II. PROBLEM STATEMENT

We consider a network of N devices embedded in the η-dimensional Euclidean space. Such a network can be represented [6] by a simplicial graph $G_{\eta,N}(\mathbf{p},\mathbf{E},\mathbf{D})$, with vertices $\mathbf{p}$, edges $\mathbf{E}=\{\varepsilon_r\}$ where r is an index for the conncection $(n,m)\in\mathbf{E}$, and weights $\mathbf{D}=\{d_{n,m}:(n,m)\in\mathbf{E}\}$. Edges represent communication links, while weights are the Euclidean distance between sensors, given by

$$d_{n,m}=\mathcal{D}([\mathbf{x}_n,\mathbf{x}_m])\triangleq\sqrt{\langle(\mathbf{x}_n-\mathbf{x}_m);(\mathbf{x}_n-\mathbf{x}_m)\rangle},\quad(1)$$

where $\mathbf{x}_n$ denotes the coordinate vector of p_n and $\langle\mathbf{x}_n;\mathbf{x}_m\rangle$ the inner product of $\mathbf{x}_n$ and $\mathbf{x}_m$, with $\mathbf{x}_n\neq\mathbf{x}_m\ \forall\ n\neq m$.

Let $\mathbf{C}$ be the binary-valued adjacent matrix of the graph $G_{\eta,N}(\mathbf{p},\mathbf{E},\mathbf{D})$, with $c_{n,m}=1$ and $c_{n,m}=0$ indicating the existence and the non-existence of a link between p_n and p_m, respectively.

The matrix $\mathbf{C}$ will be referred to as the *connectivity matrix* of the network. In a *complete* or *fully connected* graph, the associated connectivity matrix $\mathbf{C}_\text{f}=\mathbf{1}\cdot\mathbf{1}^\text{T}-\mathbf{I}$, where the superscript$^\text{T}$ denotes transpose, $\mathbf{1}$ is the all-one column vector and $\mathbf{I}$ is the identity matrix.

The term *completeness* is used in reference to the ratio ϱ of the cardinality of the set $\mathbf{E}$ and the set $\mathbf{E}_\text{f}$ of the corresponding complete graph,

$$\varrho=|\mathbf{E}|/|\mathbf{E}_\text{f}|,\quad(2)$$

where $|\cdot|$ denotes cardinality.

Likewise, the term *incompleteness* will be used in reference to the complement $1-\varrho$ of such a ratio.

The graph has the associated EDM $\mathbf{D}\triangleq[d_{n,m}]\in\mathbb{S}^{N\times N}$, where $\mathbb{S}$ denotes the set of hollow symmetric matrices. A matrix $\mathbf{S}\in\mathbb{S}^{N\times N}$ is said to be a *true* EDM in the embedding η if and only if (iff),

$$\exists\ \mathbf{X}=[\mathbf{x}_1,\cdots,\mathbf{x}_N]\in\mathbb{R}^{\eta\times N}\mid\mathbf{S}=\mathcal{D}(\mathbf{X}),\quad(3)$$

where the slightly abused notation $\mathbf{S}=\mathcal{D}(\mathbf{X})$ indicates that $s_{n,m}=\mathcal{D}([\mathbf{x}_n,\mathbf{x}_m])\ \forall\ (n,m)$.

The set of all true N-dimensional EDMs in the η-dimensional Euclidean space will be denoted by $\mathbb{E}^{N\times N}$. A matrix $\tilde{\mathbf{D}} \in \mathbb{S}^{N\times N}, \tilde{\mathbf{D}} \notin \mathbb{E}^{N\times N}$ that is close to a true EDM (in the Frobenius norm sense) will be referred as an EDM *sample*.

Let $\{\tilde{\mathbf{D}}\} = \{\tilde{\mathbf{D}}_1, \cdots, \tilde{\mathbf{D}}_K\}$ denote the set of EDM samples $\tilde{\mathbf{D}}_k$. Then, the positioning problem under consideration can be mathematically stated as follows:

$$\min_{\hat{\mathbf{X}} \in \mathbb{R}^{\eta \times N}} f(\hat{\mathbf{X}}) = ||\mathcal{G}(\{\tilde{\mathbf{D}}\}) - \mathcal{G}(\mathcal{D}(\hat{\mathbf{X}}))||_{\mathrm{F}}^2, \qquad (\mathrm{P}-1)$$

where $||\cdot||_{\mathrm{F}}$ denotes the Frobenius norm and $\mathcal{G}$ is a function to be determined.

III. Background

The first difficulty to handle in solving problem (P – 1) is to choose the function $\mathcal{G}$. If the true EDM is known, we have

$$\min_{\hat{\mathbf{X}} \in \mathbb{R}^{\eta \times N}} f(\hat{\mathbf{X}}) = ||\mathbf{D} - \mathcal{D}(\hat{\mathbf{X}})||_{\mathrm{F}}^2, \qquad (\mathrm{P}-2)$$

Problem (P – 2) is equivalent to that solved exactly by the metric MDS algorithm [2], which can be summarized as:

$$\hat{\mathbf{X}} = \left([\mathbf{U}]_{\mathrm{UL}:N\times\eta} \cdot [(\mathbf{\Lambda})^{\frac{1}{2}}]_{\mathrm{UL}:\eta\times\eta} \right)^{\mathrm{T}}, \qquad (4)$$

where $[\cdot]_{\mathrm{UL}:n\times m}$ denotes the n-by-m upper-left partition, $(\cdot)^m$ denotes element-wise (Hadamard) power of m-th order, and $\mathbf{U}$ and $\mathbf{\Lambda}$ are the eigenvector and eigenvalue matrices of the positive semi-definite Euclidean kernel defined as [7]:

$$\mathbf{K} \triangleq \mathcal{K}(\mathbf{D}) = -(\mathbf{J} \cdot (\mathbf{D})^2 \cdot \mathbf{J})/2, \qquad (5a)$$

$$\mathbf{J} \triangleq \mathbf{I} - (\mathbf{1} \cdot \mathbf{1}^{\mathrm{T}})/N. \qquad (5b)$$

A. Localization and the EDM Completion Problem

Problem (P – 2) is a trivial variation of (P – 1) and its application to source localization in wireless networks is limited, since inaccuracies of the EDM sample – in particular erasure of some entries (incompleteness) of $\{\tilde{\mathbf{D}}\}$ – have a devastating effect on positioning algorithms based on MDS. Non-surprisingly, considerable amount of work has been done on the approximation and complexion of imperfect EDM samples [8]. Traditionally, this problem is solved by non-linear optimization over an objective function designed to exploit properties of EDMs [7].

In [1], for instance, Wolkowicz *et al.* proposed to pose the EDM-AP and EDM-CP in the form:

$$\min_{\substack{\hat{\mathbf{W}} \in \mathbb{R}^{(N-1)\times(N-1)} \\ \hat{\mathbf{W}} \succeq 0}} ||\mathbf{H} \circ (\tilde{\mathbf{D}})^2 - \mathbf{H} \circ \mathcal{W}^{-1}(\hat{\mathbf{W}})||_{\mathrm{F}}^2, \qquad (\mathrm{P}-3)$$

where the $\mathbf{M} \succeq 0$ denotes positive semi-definiteness of $\mathbf{M}$, $\circ$ denotes *Hadamard* product and $\mathbf{W}$ is defined as,

$$\mathbf{W} = \mathcal{W}(\mathbf{D}) = -(\mathbf{V}^{\mathrm{T}} \cdot (\mathbf{D})^2 \cdot \mathbf{V})/2, \qquad (6a)$$

$$\mathbf{V} = \left[\begin{array}{c} -\frac{1}{\sqrt{N}}\mathbf{1} \\ \cdots\cdots\cdots\cdots\cdots \\ \mathbf{I} - \frac{1}{N+\sqrt{N}}\mathbf{1}\cdot\mathbf{1}^{\mathrm{T}} \end{array} \right]. \qquad (6b)$$

It was shown in [1] that $\mathbf{W}$ is a positive semi-definite kernel of the Euclidean distance matrix $\mathbf{D}$, that admits the unique inverse mapping $\mathbf{D} = (\mathcal{W}^{-1}(\mathbf{W}))^{\frac{1}{2}}$, where

$$\mathcal{W}^{-1}(\mathbf{W}) \triangleq \mathrm{diag}(\mathbf{W}_{\mathrm{V}}) \cdot \mathbf{1}^{\mathrm{T}} + \mathbf{1}^{\mathrm{T}} \cdot \mathrm{diag}(\mathbf{W}_{\mathrm{V}}) - 2\mathbf{W}_{\mathrm{V}} \qquad (7)$$

with $\mathbf{W}_{\mathrm{V}} = \mathbf{V} \cdot \mathbf{W} \cdot \mathbf{V}^{\mathrm{T}}$.

The weight matrix $\mathbf{H}$ in problem (P – 3) is inserted in order to control the variation of each element of the solution. To the best of our knowledge, however, no optimality theory to determine such weights exists. It is only conjectured in [1] that the entries of $\mathbf{H}$ should hold a proportion to the confidence on the corresponding entries of the given EDM sample $\tilde{\mathbf{D}}$, with $h_{n,m} = 0$ for $\tilde{d}_{n,m}$ unknown, and $h_{n,m} \gg 0$ indicating that $\tilde{d}_{n,m}$ is accurately measured. The idea behind this formulation is to exploit the positive semi-definiteness of the kernel $\mathbf{W}$ as an intrinsic constraint onto the solution. The problem can then be solved by SDP, with certain convergence advantages [1].

B. Wolkowicz's Algorithm Application

In this subsection we show an application of Wolkowicz's approach by simulating a scenario with 8 anchor nodes (AN) and 10 target nodes (TN). ANs have fixed and known position – at the corner of a cube $(10 \times 10 \times 10)$m – while TNs are uniformly distributed inside the cube. It is assumed that all nodes are ranging-capable and have limited radio connectivity, which is bounded by R_{MAX}.

The algorithm is evaluated in terms of the localization error probability $\xi(\alpha)$, which is computed from the root-mean-square (rms) positioning error $\zeta^{(\varrho)}$, given by,

$$\xi(\alpha) = P(\zeta^{(\varrho)} \leqslant \alpha), \qquad (8)$$

$$\zeta^{(\varrho)} \triangleq \left\| [\mathbf{X}]_{\mathrm{UL}:(N-A)\times\eta} - [\hat{\mathbf{X}}^{(\varrho)}]_{\mathrm{UL}:(N-A)\times\eta} \right\|_{\mathrm{F}} / \sqrt{N-A}, \qquad (9)$$

where α is a required accuracy and the coordinates of the anchor nodes are placed at the lowest A rows of $\mathbf{X}$.

Distance measurements are Gamma-distributed in accordance to the model empirically derived in [9]. The result of the simulation are shown in figure 1, parameterized by ϱ, which is set by varying R_{MAX} within the range of 2 to 20m.

It is found that in 50% of the trials, the algorithm can solve the localization problem with completeness up to 60% with an accuracy better than 3.5cm.

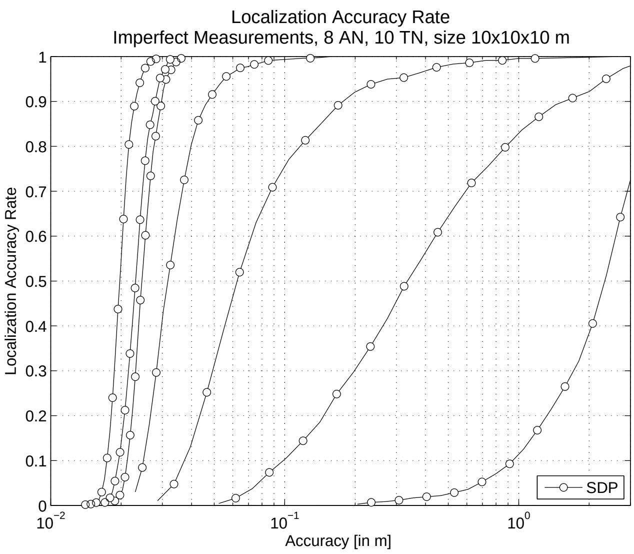


Figure 1: Performance of SDP localization algorithm with imperfect and incomplete EDMs and binary weights.

IV. Modified SDP Algorithm

Despite accurate, Wolkowicz's algorithm can be improved by utilizing a weight function that reflects the conditions under which the EDM-AP/CP is solved. For instance, the weight matrix, which in the absence of a better criterion is taken to be a binary-valued matrix related to the connectivity of the network, can be computed by exploiting the intrinsic information of the measurements and the structural properties of the associated graph. In the context of sensor localization, the problem (P – 3) can be revisited to

$$\min_{\substack{\hat{\mathbf{W}}\in\mathbb{R}^{(N-1)\times(N-1)}\\ \hat{\mathbf{W}}\succeq 0}} \left|\left|\mathcal{H}\circ\left(\bar{\mathcal{G}}^2(\{\tilde{\mathbf{D}}\})-\mathcal{W}^{-1}(\hat{\mathbf{W}})\right)\right|\right|_{\mathrm{F}}^2, \qquad (\mathrm{P}-4)$$

where $\mathcal{H}$ is function of $\{\tilde{\mathbf{D}}\}$ and $\mathbf{C}$ that controls the variation on each entry $\hat{d}_{n,m}$ in the optimization, allowing less variation for the more "important" entries and more for the others.

While the "importance" of a single entry $\hat{d}_{n,m}$ is not clearly defined, it is proportional to the accuracy of the distance estimate and the impact of the corresponding edge onto the structure of the graph. This indicates that $\mathbf{H}$ can be related to a function $\mathcal{H}_D(\{\tilde{\mathbf{D}}\})$ that measures the confidence of each $\hat{d}_{n,m}$, and to a function $\mathcal{H}_C(\mathbf{C})$ that gives the importance of that entry in the graph structure. Therefore, we define $\mathcal{H}$ as

$$\mathcal{H}(\{\tilde{\mathbf{D}}\},\mathbf{C}) \triangleq \mathcal{H}_D(\{\tilde{\mathbf{D}}\})\cdot\mathcal{H}_C(\mathbf{C}). \qquad (10)$$

A. Derivation of the Weight Function $\mathcal{H}_D$

In this subsection, the function $\mathcal{H}_D$ is derived. The confidence [10] on the estimate of the distance $d_{n,m}$ is given by the probability that its estimate $\bar{\mathcal{G}}(\tilde{d}_{n,m:1},\cdots,\tilde{d}_{n,m:K})$ is within a certain range γ from the true distance $d_{n,m}$, conditioned on the amount of offset $\rho_{n,m}$ affecting the observations $\{\tilde{d}_{n,m}\}=\{\tilde{d}_{n,m:1},\cdots,\tilde{d}_{n,m:K}\}$. Thus, we may write,

$$\mathcal{H}_D(\{\tilde{\mathbf{D}}\}) = [h_{D_{n,m}}], \qquad (11a)$$

$$h_{D_{n,m}} \triangleq P(d_{n,m}-\gamma\leqslant\bar{\mathcal{G}}(\{\tilde{d}_{n,m}\})\leqslant d_{n,m}+\gamma|\rho_{n,m}). \qquad (11b)$$

Several solutions for the function $\bar{\mathcal{G}}$ were investigated in [9]. It was found that the best-performing and lest complex alternative is to average the EDM samples as shown below

$$\bar{\mathcal{G}}(\{\tilde{\mathbf{D}}\}) = \bar{\tilde{\mathbf{D}}} = [\bar{\tilde{d}}_{n,m}], \qquad (12a)$$

$$\bar{\tilde{d}}_{n,m} \triangleq \begin{cases} \dfrac{1}{K_{n,m}}\sum\limits_{k=1}^{K_{n,m}}\tilde{d}_{n,m:k} & \text{if } K_{n,m}\geqslant 1 \\ 0 & \text{if } K_{n,m}=0 \end{cases},$$

where $K_{n,m}$ is the number of samples available for $d_{n,m}$.

Since any arbitrary pair of nodes can be considered when computing equation (11b), the subscripts n and m will be henceforth omitted, without loss of generality, when introducing the following assumptions on the random $\tilde{d}_{n,m}$:

i) $\mathrm{E}[\tilde{d}] = d+\rho$;

ii) $\mathrm{E}[(\tilde{d}-\mathrm{E}[\tilde{d}])^2] = \sigma^2_{\tilde{d}}$;

iii) $\mathrm{E}[\tilde{d}_k\cdot\tilde{d}_q] = \mathrm{E}[\tilde{d}_k]\cdot\mathrm{E}[\tilde{d}_q]$,

where the subscripts k and q indicate distinct samples of $\tilde{d}$.

Notice that these assumptions are not highly restrictive and rather intuitive within the context of this work.

Now consider the variable $\epsilon = \bar{\tilde{d}}$. Due to the Central Limit Theorem, the distribution of ϵ approaches the normal (Gaussian) distribution $\mathcal{N}(\bar{\epsilon},\sigma^2_\epsilon)$, regardless of the distribution of the measurements $\tilde{d}$, provided that K is sufficiently large. Substituting the normal distribution into equation (11b) we obtain, after some algebra,

$$P(d-\gamma\leqslant\epsilon\leqslant d+\gamma|\rho)=\frac{1}{\sqrt{\pi}}\int_{l_1}^{\infty}e^{-x^2}dx-\frac{1}{\sqrt{\pi}}\int_{l_2}^{\infty}e^{-x^2}dx, \qquad (13)$$

where $l_1=(d-\gamma-\bar{\epsilon})/(\sigma_\epsilon\sqrt{2})$ and $l_2=(d+\gamma-\bar{\epsilon})/(\sigma_\epsilon\sqrt{2})$.

An immediate consequence of assumption i is that

$$\bar{\epsilon} \triangleq \mathrm{E}[\epsilon] = \mathrm{E}[\tilde{d}] = d+\rho. \qquad (14)$$

The variance of ϵ can be computed as follows. From the definition, we have,

$$\sigma^2_\epsilon \triangleq \mathrm{E}[(\epsilon-\bar{\epsilon})^2] = \mathrm{E}\left[\left(\frac{1}{K}\sum_{k=1}^{K}\tilde{d}_k-\mathrm{E}[\tilde{d}]\right)^2\right] \qquad (15)$$

$$=\frac{1}{K^2}\cdot\mathrm{E}\left[\left(\sum_{k=1}^{K}\tilde{d}_k-K\cdot\mathrm{E}[\tilde{d}]\right)^2\right]=\frac{1}{K^2}\cdot\mathrm{E}\left[\sum_{k=1}^{K}\left(\tilde{d}_k-\mathrm{E}[\tilde{d}]\right)^2\right]+$$

$$+\frac{1}{K^2}\cdot\mathrm{E}\left[\sum_{k=1}^{K}\sum_{q\neq k}^{K}\left(\tilde{d}_k-\mathrm{E}[\tilde{d}]\right)\cdot\left(\tilde{d}_q-\mathrm{E}[\tilde{d}]\right)\right].$$

Next, recognize that the expectation on the rightmost term of the second line of equation (15) is the variance of $\tilde{d}$, as stated in assumption ii, which yields,

$$\sigma^2_\epsilon = \frac{\sigma^2_{\tilde{d}}}{K}+\frac{1}{K^2}\times \qquad (16)$$

$$\times\sum_{k=1}^{K}\sum_{q\neq k}^{K}\left(\mathrm{E}[\tilde{d}_k]\mathrm{E}[\tilde{d}_q]-\mathrm{E}[\tilde{d}_k]\mathrm{E}[\tilde{d}]-\mathrm{E}[\tilde{d}_q]\mathrm{E}[\tilde{d}]+\mathrm{E}[\tilde{d}]^2\right)=\frac{\sigma^2_{\tilde{d}}}{K},$$

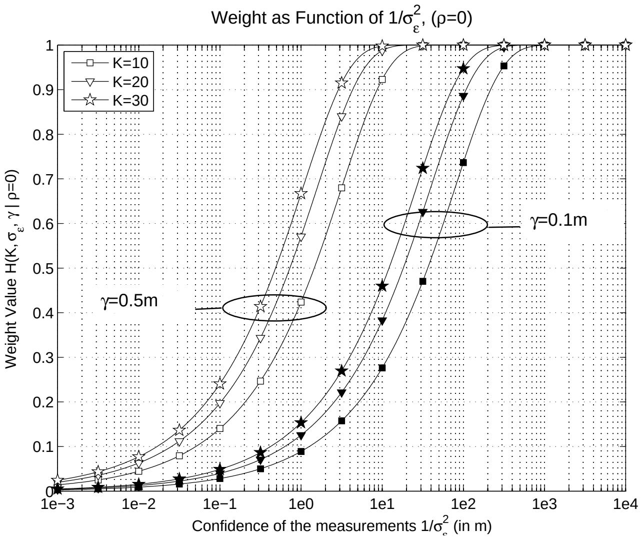
where the double summation vanishes due to the independence of $\tilde{d}_k$ and $\tilde{d}_q$, as per assumption iii.

Substituting these results into equation (13), and returning to the original notation we finally obtain

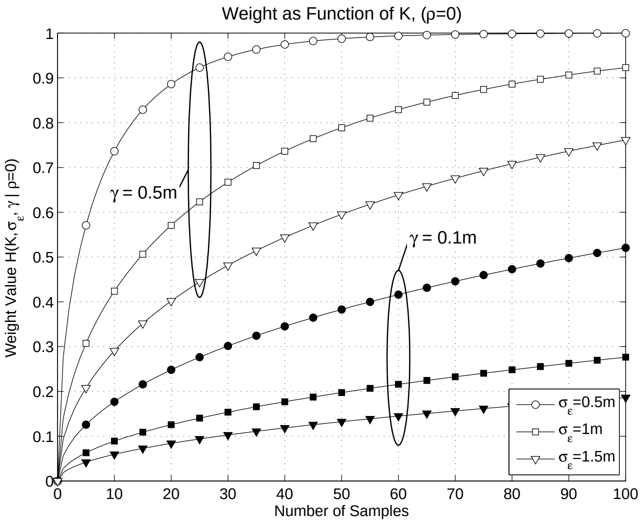
$$h_{n,m}\approx Q\left(-\frac{(\gamma+\rho_{n,m})\cdot\sqrt{K_{n,m}}}{2\cdot\hat{\sigma}_{\tilde{d}_{n,m}}}\right)-Q\left(\frac{(\gamma-\rho_{n,m})\cdot\sqrt{K_{n,m}}}{2\cdot\hat{\sigma}_{\tilde{d}_{n,m}}}\right), \qquad (17)$$

where $\hat{\sigma}^2_{\tilde{d}_{n,m}}$ is the sample variance of $\tilde{d}_{n,m}$, computed from the available samples, and $Q(x)$ is the Gaussian Q-function.

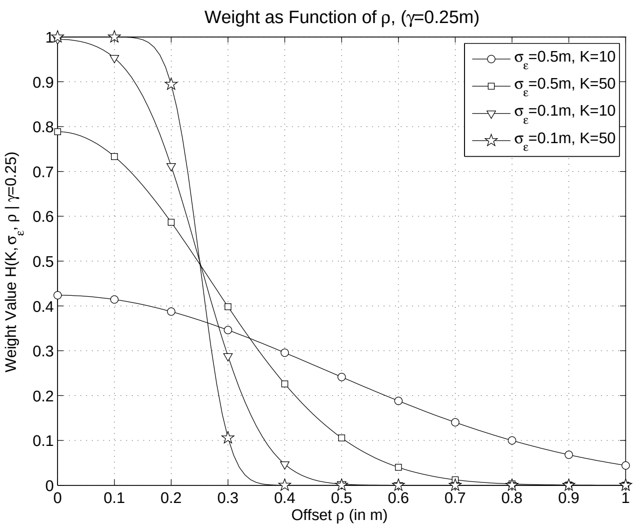
Plots of the weight function $\mathcal{H}_D$ are shown in figure 2. Notice that $\mathcal{H}_D$ can only be calculated if the offset $\rho_{n,m}$ is known, as indicated by the conditionality sign there inserted. While offsets cannot be estimated form the measurements $\tilde{d}_{n,m}$ alone, these parameters can be estimated collectively, after a group of nodes have been localized. This problem is, however, beyond the scope of this work and will be dealt with in a future article. For now, we simply utilize the estimate $\hat{\rho}_{n,m} = 0\ \forall\ (n,m)$ into equation (17), although the effect of non-zero offsets are included in our simulations (see section V).



(a) Influence of the sample variance.



(b) Influence of the sample size.



(c) Influence of offset.

Figure 2: Weight function against: (a) The confidence on the distance measurements; (b) The sample size; (c) The offset.

B. *Derivation of the Weight Function* $\mathcal{H}_C$

In this subsection, the function $\mathcal{H}_C$ is derived. Let us start by recalling the role of $\mathcal{H}_C$ in the proposed localization algorithm as a whole. Just as $\mathcal{H}_D$ relates exclusively to the accuracy of the pairwise distances estimates, being independent of the structure of the graph describing the network, $\mathcal{H}_C$ should relate to the impact that the presence/absence of each edge ε_r has on the overall structure of the graph, regardless of how good or bad the corresponding distance is estimated. In other words, the (n, m)-th entry of $\mathcal{H}_C$ should be a metric of the relevance that the edge ε_r has to the problem of resolving the graph.

The first challenge we face is to find an adequate way to assign a numerical value to such a metric, relying solely on connectivity information, $i.e.$, using nothing but $\mathbf{C}$.

It has been recently shown that the structural property of a graph is strongly correlated with the eigenspectra of its representation matrices which are computed directly from the adjacent matrix [11]. In light of those results, we conjecture that the relevance of a particular edge ε_r can be estimated by measuring the amount of *perturbation* that the deletion of the connection (n, m) has on the spectrum of the graph. The relative perturbation of the spectra graph [11] can be measured as

$$\delta \triangleq \frac{\|\mathbf{\Lambda}_G - \mathbf{\Lambda}_{G_p}\|_2}{\|\mathbf{\Lambda}_G\|_2}, \quad (18)$$

where $\|\cdot\|_2$ denotes the norm-2 and $\mathbf{\Lambda}_G$ and $\mathbf{\Lambda}_{G_p}$ are the vectors of eigenvalues of the representation matrices of the non-perturbed and perturbed graph G, respectively.

Several different graph representation matrices were considered in [11], with similar results. Any of such matrices can be used for the purpose of computing $\mathcal{H}_C$. In this work we use, for simplicity, the *signless Laplacian* matrix

$$\mathbf{L} \triangleq \mathbf{\Theta} + \mathbf{C}, \quad (19)$$

where $\mathbf{\Theta}$ is a diagonal matrix whose entries are given by the *degree* of the node p_m, $i.e.$ the number of connections that node has with other nodes in the network.

From the above, we finally arrive at

$$\mathcal{H}_C(\mathbf{C}) = [h_{C_{n,m}}], \quad (20a)$$

$$h_{C_{n,m}} \triangleq \frac{\delta_r}{\max(\mathbf{\Delta})}, \quad (20b)$$

where $\mathbf{\Delta} = \{\delta_1, \cdots, \delta_{|\mathbf{E}|}\}$.

V. Simulation Results and Comparisons

In this section the performance of the modified Wolkovicz's algorithm is studied through computer simulations. To this end, we considered the same scenario as described in section III,with a connectivity range of $R_{\text{MAX}} = 9.7$m, which results in a completeness of about 70%. Distance estimates were randomly generated following a Gamma-distribution in accordance with experimental results observed using time-of-arrival (ToA) based ultra-wideband (UWB) radios [9]. In order to emulate the fact that the UWB channel changes for each pair of nodes, however, we allow the standard deviation of each distance estimate $\tilde{d}_{n,m}$ to vary randomly from 0.1 to 3m, independently of the nominal value $d_{n,m}$.

Different topology realizations are tested, each solved by using either a binary weight function ($\mathbf{H}$), the weight function ($\mathcal{H}_D$) only, the weight function ($\mathcal{H}_C$) only or, finally, a the complete weight function $\mathcal{H} = \mathcal{H}_D \cdot \mathcal{H}_C$.

The performance of the localization algorithm with different weight functions is compared in terms of their localization error relative to and normalized by the corresponding error achieved when binary weights are used. Mathematically, this figure is given by the ratio

$$\psi = (\zeta_{\mathrm{b}}^{(70)} - \zeta_{\mathrm{w}}^{(70)})/\zeta_{\mathrm{b}}^{(70)}, \tag{21}$$

where the subscript b and w indicate binary and non-binary (weighted) $\mathbf{H}$ matrices, respectively.

The results of the comparison are given in figure 3, which shows the probability density function (pdf) of the relative gain ψ achieved with different parameters γ.

It can be noticed that the relative gain distribution is centered slightly to the left of the origin. It is also found, however, that the distributions exhibit long tails. This indicates that, although binary weights lead to better results than non-binary weights, the significant performance improvement can be achieved if an appropriate criterion can be found to estimate whether the utilization of non-binary weights is beneficial or not. This might be achieved in certain applications, where the presence of a sufficiently large number of anchors can be used to extract statistical sample of the entire network.

However, in more realistic scenarios where the measurements have complete different statistics (due to device heterogeneity, environment, etc.) and a data diversity exists, the benefits of the usage of weights can be easily proved by simulations.

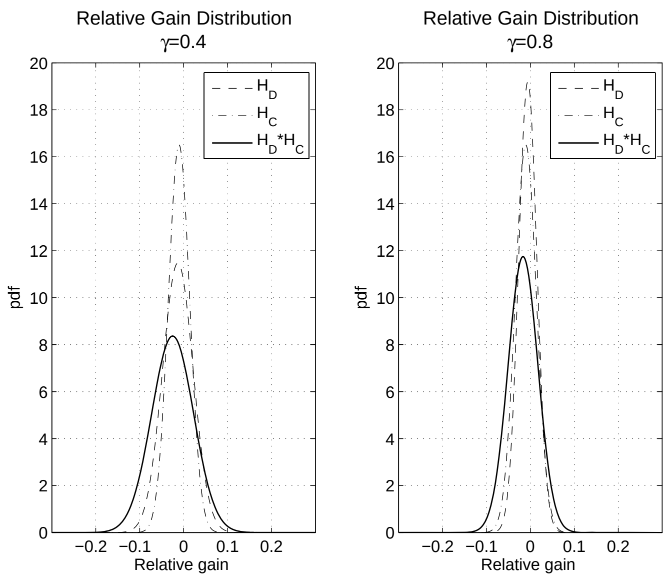


Figure 3: Distribution of the relative ψ at $\varrho = 70\%$

In general, however, the behavior of the relative error distribution shown in figure 3 suggests that the effectiveness of utilizing non-binary weights is conditioned on yet other factors not captured by the functions $\mathcal{H}_D$ and $\mathcal{H}_C$, for instance the network diversity.

VI. Conclusions

In this paper, source localization from imperfect and incomplete range information is considered. We focus on a technique employing a combination of MDS and EDM-AP/CP using semi-definite programming. The EDM-AP/CP technique admits a weight matrix in the cost-function to be optimized, which is meant to control the amount of variation that each entry undergoes during the optimization. Our contribution is the study of an adequate formula for such a weight function, taken into account the conditions of the considered sensor network application.

Two components of the weight matrix are identified and quantified, namely, a weigh function based on confidence-bound statistics of the distance estimates, and another based on spectral properties of graph representation matrices.

Computer simulations show that significant improvement in the localization accuracy can be achieved by utilizing the weight function derived, provided that an adequate criterion for the selection of the weights can be found.

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