DISTRIBUTED GABBA SPACE-TIME CODES WITH COMPLEX SIGNAL CONSTELLATIONS

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ABSTRACT

This paper is on the design of practical distributed space-time codes for wireless relay networks. The amplify-and-forward (AF) scheme is used in a way that each relay transmits a scaled version of the linear combination of the received symbols. We propose distributed GABBA codes, which is generalized to any number of relays and independent of modulation type. Using linear orthogonal decoding in the destination makes it feasible to employ large number of potential relays to improve diversity order.

1. INTRODUCTION

In [1], a cooperative strategy was proposed which achieves a diversity factor of *R* in a *R*-relay wireless network, using the so called distributed space-time codes (DSTC). In this strategy, a two-phase protocol is used. In phase one, the transmitter sends the information signal to the relays and in phase two, relays send information to the receiver. The signal sent by every relay in the second phase is designed as a linear function of its received signal. It was shown that the relays can generate a linear space-time codeword at the receiver, as in a multiple antenna system, although they only cooperate distributively. This method does not require decoding at the relays and for high SNR it achieves the optimal diversity factor [1]. Although distributed space-time coding does not need instantaneous channel information in the relays, it requires full channel information at the receiver, $i.e.$, both the channel from the transmitter to relays and the channel from relays to the receiver, need be known at the receiver. This requires that training symbols be sent from both the transmitter and relays.

Recently, the design of practical DSTC, that lead to reliable communication in wireless relay networks, has been presented in [2] and [3]. In this paper, we focus on the space-time cooperation using generalized ABBA (GABBA) codes [4], which are systematically constructed, orthogonally decodable, full-rate, full-diversity space-time block codes. In [3], we show that GABBA codes with real modulation are applicable as distributed space-time codes based on [1] in which relays transmit the linear combinations of the scaled version of received signals. In this paper, we propose a complex DSTC design, employing GABBA codes, in which any two-dimensional signal constellations can be used.

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2. SYSTEM MODEL

Consider a network consisting of a source denoted *s*, one or more relays denoted $r = 1, 2, \ldots, R$, and one destination denoted *d*. It is assumed that each node is equipped with a single antenna. We denote the source-to-*r*th relay, and *r*th relay-to-destination links by f_r and g_r, respectively. Suppose each link has Rayleigh fading, independent of the others. Therefore, f_r and g_r are i.i.d. complex Gaussian random variables with zero-mean and variances σ_f^2 and σ_g^2, respectively. Similar to [1], our scheme requires two phase of transmission. During the first phase, the source node s transmits a signal $\boldsymbol{s} = [s_1, \ldots, s_T]^T$, consisting of T symbols to *all* relays. We assume the following normalization, $E\{\boldsymbol{s}^H\boldsymbol{s}\} = 1$. Thus, from time 1 to *T*, signals $\sqrt{P_1 T}s_1, \ldots, \sqrt{P_1 T}s_T$ are sent to all relays by the source. The average total transmitted energy in *T* intervals is $P_1 T$. Assuming f_r is not varying during *T* successive intervals, the received $T \times 1$ signal at the *r*th relay can be written as

$$\boldsymbol{y}_r = \sqrt{P_1 T} f_r \boldsymbol{s} + \boldsymbol{v}_r, \tag{1}$$

where $\boldsymbol{v}_r$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with the variance of N_1.

When *complex constellation* is used, the received signal at the destination in the second phase can be written as [2]

$$\boldsymbol{y} = \sum_{r=1}^{R} g_r \left(\rho_r^* \boldsymbol{A}_r \boldsymbol{y}_r + \rho_r \boldsymbol{B}_r \boldsymbol{y}_r^*\right) + \boldsymbol{w}, \tag{2}$$

where $\boldsymbol{A}_r$ and $\boldsymbol{B}_r$ can be obtained by representing the *r*th column of the $T \times R$ dimensional distributed space-time code matrix $\boldsymbol{S}$ as $\boldsymbol{c}_r = \boldsymbol{A}_r \boldsymbol{s} + \boldsymbol{B}_r \boldsymbol{s}^*$, and ρ_r is the scaling factor at relay r. When there is no instantaneous channel state information (ICSI) at the relays, but statistical channel state information (SCSI) is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is,

$$\rho_r^2 = \frac{P_2}{\sigma_f^2 P_1 + N_1}, \tag{3}$$

where P_2 is the average transmitted power from relay r.

From (1)-(3), the received signal can be calculated to be

$$\boldsymbol{y} = \sqrt{\frac{P_1 P_2 T}{\sigma_f^2 P_1 + N_1}} \boldsymbol{S}\boldsymbol{h} + \boldsymbol{w}_T, \tag{4}$$

where the distributed space-time code matrix $\boldsymbol{S}$ should be appropriately designed. The $R\times 1$ vector $\boldsymbol{h}$ is the equivalent channel, which its rth component is

$$h_r = \begin{cases} f_r g_r, & \text{if } \boldsymbol{c}_r \in \Omega, \\ f_r^* g_r, & \text{if } \boldsymbol{c}_r \in \Omega^*, \end{cases} \tag{5}$$

where $\boldsymbol{c}_r$ is the rth column of the code matrix $\boldsymbol{S}$, and sets Ω and Ω^* are defined as sets of columns of an appropriate code matrix that are consisting of $s_1, \dots, s_T$ and $s_1^*, \dots, s_T^*$, respectively. That is

$$\Omega = \{\boldsymbol{c}_k | \, c_{k,l} \in \{\pm s_l\}, \, k, l = 1, \dots, T\}, \tag{6}$$

$$\Omega^* = \{\boldsymbol{c}_k | \, c_{k,l} \in \{\pm s_l^*\}, \, k, l = 1, \dots, T\}. \tag{7}$$

In order to write the received signal in the form of (4), it is necessary that the r-th column of the code matrix contains the conjugates $s_1^*, \dots, s_T^*$ only or information symbol $s_1, \dots, s_T$ only. In other words, the combining matrix at the r-th relay, $\boldsymbol{D}_r$, which is corresponding to the r-th column of the code matrix $\boldsymbol{S}$, satisfy the following condition

$$\boldsymbol{D}_r = \begin{cases} \boldsymbol{A}_r, & \text{if } \boldsymbol{c}_r \in \Omega, \\ \boldsymbol{B}_r, & \text{if } \boldsymbol{c}_r \in \Omega^*. \end{cases} \tag{8}$$

3. COMPLEX DSTC DESIGN WITH DISTRIBUTED GABBA CODES

In complex orthogonal design, we are going to apply the designs which have full-rate, i.e., $1/2$, due to the two-phase transmission nature of the system, and whose columns are composed by either the information symbols $s_1, \dots, s_T$, or their conjugate $s_1^*, \dots, s_T^*$, exclusively. Similar to Alamouti design [5] and quasi-orthogonal design [6] investigated in [2], in which the transpose of space-time matrix is used, the transposed of GABBA codes can be used. This is because the fact that the transpose of $T \times T$ GABBA mother codes has a structure of $\boldsymbol{S} = [\boldsymbol{G}_1 \, \boldsymbol{G}_2^*]$, where $\boldsymbol{G}_1$ and $\boldsymbol{G}_2^*$ are $T \times T/2$ matrices, which are the function of information symbols and their conjugates, respectively. Therefore, transpose of GABBA codes can be applied in AF DSTC. Furthermore, using distributed GABBA codes, the sets Ω and Ω^* in (6)-(7) can be rewritten as

$$\Omega = \left\{ \boldsymbol{c}_k | \, k = 1, \dots, \frac{T}{2} \right\}, \; \Omega^* = \left\{ \boldsymbol{c}_k | \, k = \frac{T}{2} + 1, \dots, T \right\}. \tag{9}$$

3.1. Code Construction

In this subsection, we propose complex distributed GABBA designs to construct DSTC that achieve full-diversity and full-rate. We define a complex distributed GABBA design of the size $T \times R$, where T is power of two, and R is the number of relays which can be $R \leq T$, as the submatrix of transpose of GABBA codes.The distributed GABBA codes can be constructed by simply removing arbitrary $T-R$ columns of the transpose of $T \times T$ GABBA codes.

As an example, we explain the application of the 2×2, 4×4, and 8×8 GABBA codes. Moreover, since every symbols appears only once in each column, which is true for all GABBA codes, $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ has the structure of a matrix whose entries can be 1, 0, or -1.

The 2×2 distributed GABBA code matrix is

$$\boldsymbol{S} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}. \tag{10}$$

The matrices used at the relays are

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \boldsymbol{B}_1 = \boldsymbol{A}_2 = \boldsymbol{0}_2 \text{, and } \boldsymbol{B}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{11}$$

The 4×4 distributed GABBA code matrix is given by

$$\boldsymbol{S} = \begin{bmatrix} s_1 & -s_2 & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1^* & s_2^* \\ s_4 & s_3 & -s_2^* & s_1^* \end{bmatrix}. \tag{12}$$

The matrices used at the relays are

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{B}_1 = \boldsymbol{0}_4,$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \boldsymbol{B}_2 = \boldsymbol{0}_4,$$

$$\boldsymbol{A}_3 = \boldsymbol{0}_4, \, \boldsymbol{B}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$A_4 = \mathbf{0}_4,\ B_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \quad (13)$$

It is easy to see that A_i and B_i in (11) and (13) are either zero or unitary.

The 8×8 distributed GABBA code matrix is given by

$$S = \begin{bmatrix} s_1 & -s_2 & -s_3 & s_4 & -s_5^* & -s_6^* & -s_7^* & -s_8^* \\ s_2 & s_1 & -s_4 & -s_3 & s_6^* & -s_5^* & s_8^* & -s_7^* \\ s_3 & -s_4 & s_1 & -s_2 & s_7^* & s_8^* & -s_5^* & -s_6^* \\ s_4 & s_3 & s_2 & s_1 & -s_8^* & s_7^* & s_6^* & -s_5^* \\ s_5 & -s_6 & -s_7 & s_8 & s_1^* & s_2^* & s_3^* & s_4^* \\ s_6 & s_5 & -s_8 & -s_7 & -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_7 & -s_8 & s_5 & -s_6 & -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_8 & s_7 & s_6 & s_5 & s_4^* & -s_3^* & -s_2^* & s_1^* \end{bmatrix}. \quad (14)$$

Since A_i corresponds to the ith column of the code matrix S, one can easily construct matrices $\{A_1, A_2, A_3, A_4\}$ and $\{B_5, B_6, B_7, B_8\}$ used at the relays. For higher dimension GABBA mother codes, like 16×16 and 32×32 GABBA codes, the same characteristic holds, because of the structure of $S = [G_1\, G_2^*]$. But due to lack of space, we omit to write the corresponding S, A_i, and B_is.

3.2. The Decoding Algorithm

For the case of 2×2 distributed GABBA codes, the orthogonal decoder is equivalent to ML decoder, like Alamouti code. Similar to decoding formulas in [7, Eq. (4.79)], the maximum-likelihood decoding can be applied for the decoding of distributed GABBA codes, when $R > 2$. Moreover, since the log-likelihood ratio cost function in the $T \times R$ distributed GABBA code can be decomposed to two separate function, pairwise maximum-likelihood (PW ML) decoding [7] is equivalent to ML decoding, similar to quasi-orthogonal (QO) DSTC.

3.2.1. GABBA Linear Orthogonal Decoder

The complex decoding techniques, like maximum likelihood (ML) and pairwise ML [7], prevent applicability of the code matrices with large dimensions. Using distributed GABBA codes with linear orthogonal decoding, we can employ large number of potential relays to improve diversity order. Therefore, instead of complex ML decoding for general full-rate, full-diversity space-time codes or pairwise ML decoding, we can use GABBA linear orthogonal decoding for full-rate, full-diversity $T \times R$ GABBA codes. In the case of distributed GABBA codes, the same orthogonal decoder presented in [4] can be used. But the output of the decoder should be passed again to GABBA encoder. The first column of the output of GABBA encoder is corresponding to the recovered signal.

3.3. Enhanced Distributed GABBA Codes

The GABBA orthogonal decoder suboptimally separates non orthogonal columns of the matrix G_1, i.e. $c_r \in \Omega$, and G_2, i.e., $c_r \in \Omega^*$. However, due to the orthogonality of the columns in G_1 and G_2, the performance of the $T \times R$ distributed GABBA codes can be improved when $1 < R < T-1$. The relays are divided into two categories. We should allocate almost half of the relays with $c_i \in \Omega$, and the remaining relays should be corresponding to $c_i \in \Omega^*$. In this scheme, we put the ith column of code matrix S, i.e., c_i, which is corresponding to A_i and B_i in the rth relay. One possible choice of the index i which satisfies the condition stated above is

$$i(r) = \begin{cases} r, & \text{if } r \leq \lceil R/2 \rceil, \\ T - R + r, & \text{if } r > \lceil R/2 \rceil. \end{cases} \quad (15)$$

Thus, (8) will be changed to

$$D_r = \begin{cases} A_i(r), & \text{if } r \leq \lceil R/2 \rceil, \\ B_i(r), & \text{if } r > \lceil R/2 \rceil, \end{cases} \quad (16)$$

for $r = 1, \ldots, R$. In the case of fixed relay scenario in which all R deployed relays are used for relaying the transmitter's data, we simply assign the index of the appropriate column to the rth relay. In the case of dynamic relay scenario, in which the number of employing relays are varying depending on channel conditions, some control bits should be exchanged between the relays and the destination to inform them of the appropriate index i at the rth relay.

4. SIMULATION RESULTS

In this section, the performance of distributed GABBA space time codes are studied through simulations. The signal symbols are modulated as QPSK. However, any two-dimensional constellations can be exploited. We use equal power allocation in the two phases and also among the relays. Assume the relays and the destination have the same value of noise power and all the links have unit-variance Rayleigh flat fading.

Fig. 1 compares the performance of distributed GABBA space-time codes with the indexing format in (8) and Enhanced Distributed GABBA codes presented in Subsection 3.3, when orthogonal decoding is used. The coherence time of network channels is such that $T = 8$, and the number of relays is assumed to be $R = 2, 3, \ldots, 6$. In this case, the best choice of GABBA codes is the 8×8 GABBA mother code, shown in (14). Then, A_i and B_i corresponding to the ith column of (14) is set to an appropriate relay. If $R < 8$, then $8 - R$ arbitrarily selected columns in (14) are deleted. Since distributed GABBA codes and enhanced distributed GABBA codes have the same performance when $R = 1, 7, 8$, we do not show them in Fig. 1. One can observe that at BER 10^{-2}, a gain of 7 dB is obtained using

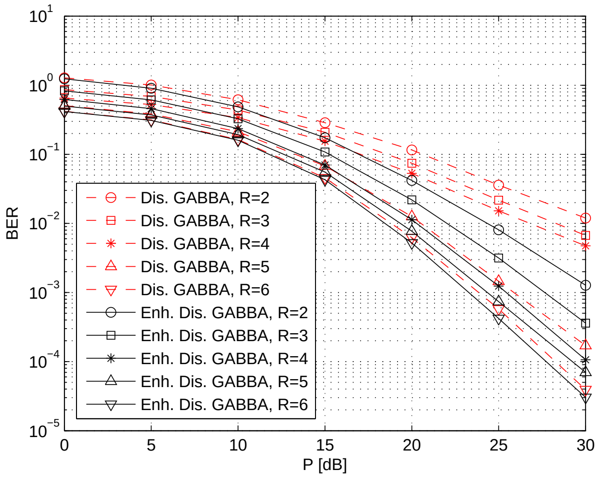

Fig. 1. The average BER curves of relay networks employing distributed GABBA space-time codes and their enhanced version with QPSK signals and using linear orthogonal decoder.

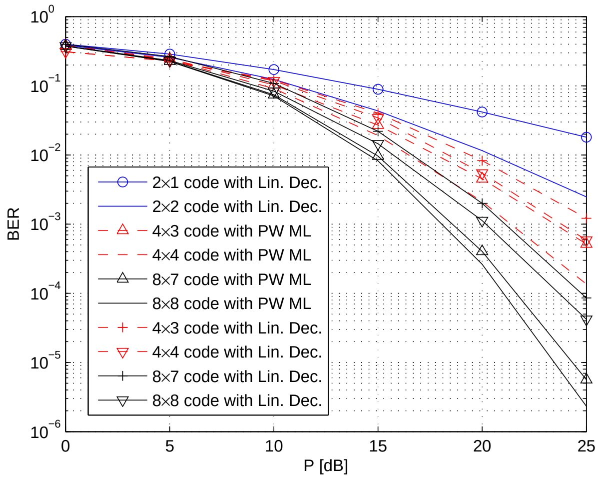

Fig. 2. The average BER curves versus SNR of relay networks employing distributed space-time codes using different decoding techniques, with QPSK signals and $T = 8$.

enhanced distributed GABBA codes, when $R = 4$. On the other hand, the linear decidability of such codes allow us to increase the scale (R) of the cooperative network in feasible wireless systems, such as wireless sensor networks.

In Fig. 2, the BERs of the system with different DSTCs and decoding techniques are considered. It can be seen in a network with 1 to 8 relays, diversities about 1 to 8 are achieved. In the case of $R \leq 2$, 2×2 GABBA mother code, or transpose of Alamouti code as stated in (10), can be employed. Note that the linear orthogonal decoder is equivalent to ML decoder, in this case. When $2 < R \leq 4$, the 4×4 GABBA mother code, which stated in (12), is an appropriate choice. For this case, we applied both GABBA linear decoder and PW ML. It can be seen that the 4×4 code with PW ML outperforms from the same code using linear decoder by about 3 dB at 10^{-3}. Finally, for $R > 4$, a suitable choice for DSTC could be the 8×8 GABBA mother code, given in (14). Note that $T \times R$ DSTC codes can be constructed by simply removing $T - R$ columns of $T \times T$ general DSTC code. Fig. 2 demonstrates that the 8×8 code with PW ML outperforms from the same code using linear decoder by about 3 dB at 10^{-4}. It is important to note that GABBA DSTC and rotated QO DSTC (see, e.g., [7] and [2]) with PW ML decoding have exactly the same performance, as we have studied in [3]. Moreover, it can be seen that GABBA DSTCs with linear orthogonal decoding have some performance loss, in expense of simple decoding, although they achieve the same diversity order as GABBA DSTCs with PW ML decoding.

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