

# Clusterization for Robust Geographic Routing in Wireless Sensor Networks

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**Abstract**—A cross-layer algorithm for geographic routing in Wireless Sensor Networks (WSNs) is proposed, which is robust to dead-ends and resilient to topological variations due to network dynamics. The solution combines ideas of network tessellation (clusterization) with greedy forwarding, without suffering from the problems afflicting landmark-based alternatives. The clusterization algorithm is based on a recently-discovered graph-spectral property and relies on connectivity information only. Cluster sizes can be varied, allowing for different trade-offs between Packet Delivery Success Ratio (PDSR) and Average Packet Delivery Latency (APDL) to be reached. Simulation results show that the technique can substantially improve the PDSR in networks where large concave holes (dead-ends) are present, with no or little impact on APDL.

## I. INTRODUCTION

In Wireless Sensor Networks (WSNs), especially those of larger scale employing hundreds or thousands of sensors (nodes), information exchange between sensors separated by distances exceeding radio range requires packets to be routed towards their destinations through chains of relays [1]. Unlike other large-scale systems such as cellular and computer networks, where routing can be aided by tables maintained in certain nodes (routers), WSNs often require reactive routing protocols that can cope with the constant topological changes resulting, *e.g.*, from nodal duty-cycles and variations in the radio range due to weather conditions [2].

When nodes have knowledge of their location relative to one another, even if not exact [3], greedy geographic routing may be an effective technique to autonomously and efficiently route packets through the network [4]. Unfortunately, a well-known problem of simple greedy geographic routing is that packets may be lost if anywhere along the multi-hop chain a relay in the forwarding direction cannot be found. This dead-end problem has prompted the proposal of several clever mitigation techniques, with approaches ranging from purely autonomous [5], to centralized<sup>1</sup> methods such as graph planarization [4].

The robustness associated with purely autonomous dead-end bypassing techniques may, however, come at the expense of efficiency, since the discovery of alternative routes in the presence of a dead-end often require excessive back-and-forth signalling. Indeed, although an autonomous method is a sensible choice in networks with volatile topologies, in many cases global topological features of networks are stable enough to be exploited by routing protocols.

<sup>1</sup>Reference [6] was brought to our attention by the reviewers. In that work a distributed planarization technique is proposed. In any case, planarization techniques even if implemented in a distributed fashion, are unlikely to be resilient to topological variations due to network dynamics.

In [7], for instance, an interesting idea that combines the advantages of static and autonomous routing approaches was proposed. The method thereby relies on a pre-processing step where the entire network is partitioned into combinatorial Voronoi tiles centered at landmarks, in conjunction with local (autonomous) routing procedures. In other words, packets are forwarded greedily within tiles, and following a global routing table from tile to tile [7]. It is found that if landmarks are properly selected so as to capture global topological features, the Delaunay graph that interconnects such landmarks can be used to effectively guide global routing across tiles.

A main weakness of landmark-based schemes, however, is that landmarks are typically selected manually (heuristically) or randomly [7], [8]. This is particularly limiting when considered that the performance of such routing schemes is severely degraded by bad landmark selections [9].

In this article we propose an alternative to landmark-based geographic routing for WSNs, that retains the advantages of the latter – namely, robustness to dead-ends – while avoiding the landmark selection problem altogether. We start by noticing that the underlying innovation introduced by landmark-based techniques is not landmarking in itself, but the associated tessellation of the network into tiles of nodes strongly connected within each tile. Furthermore, we observe that the notion of a Delaunay graph interconnecting all of the network tiles overshadows the more fundamental notion of adjacent tile connectivity.

These two principles: that tiles are made up of strongly interconnected nodes; and that nodes at the boundaries of adjacent tiles are also interconnected, are what improve the probability of success of autonomous packet-relaying within tiles, while ensuring global reach via a succession of “tile-to-tile hops”.

The underlying concepts of our technique sprout from the arguments above and can be summarized as follows. First, we propose to tessellate (or clusterize) the network *directly* according to its connectivity. The clusterization algorithm here-proposed relies on a recently-discovered spectral property of the connectivity matrix of the graph representing the network and is briefly described in section II-A. Unlike [7], [9], the cluster-to-cluster forwarding protocol is based only on the knowledge of connectivity amongst adjacent clusters and is described in section II-B.1. The routing scheme used inside each cluster is a simple greedy forwarding algorithm and is briefly reviewed in section II-B.2. Simulation results are given and discussed in section III, and concluding remarks are given in section IV.

## II. SOLUTION DESCRIPTION

This section provides a detailed description of the cluster-based geographic routing protocol that is mainly targeted on large-scale WSNs. The proposed position-centric routing protocol is composed of two operational parts: global routing among clusters and localized relay-selection inside clusters. The former, which corresponds to the main contribution of this work, exploits network graph spectral analysis to divide network into clusters composed of sufficiently connected nodes; while the latter deals with localized relay-selection inside clusters using (not necessarily, but for the time being) simple greedy forwarding.

### A. Hierarchical Clusterization Procedure

In this subsection we briefly review the clusterization method first introduced in [10]. This clusterization procedure is a graph-spectrum mathematical tool employed to divide the network into sub-groups, named clusters, whose constituent nodes are sufficiently connected to one another.

The clusterization method relies on the graph-theoretical spectra analysis of the network graph. Specifically, clusters are identified by evaluating the eigenvalues and eigenvector components of the Laplacian matrix associated to the meshed network graph [11]. The method progressively decomposes the overall network into sub-clusters using as stop criteria either the minimum completeness or the compound number of nodes. For details please refer to [10].

Initially, to generate the eigenvalue spectra of the network graph  $\mathcal{G}$ , the Connectivity matrix  $C$ , is converted into the Laplacian matrix  $L$ , as per equation (1),

$$L = M - C, \quad (1)$$

where  $M$  (a.k.a. the Node-degree matrix) is a diagonal matrix whose  $i$ th diagonal element is the number of connections the corresponding node has with its neighbors [10].

In what follows, the  $k$ th iteration of any procedure will be denoted by a superscript  $(k)$ . The second smallest eigenvector  $\mathbf{b}^{(k)}$  of the Laplacian matrix computed at the  $k$ th iteration is partitioned into two complementary parts as follows,

$$\begin{cases} \{b_i^{(k)}\} &= \{b_r^{(k)} \mid b_r^{(k)} \leq \gamma^{(k+1)}\}, \\ \{b_j^{(k)}\} &= \mathbf{b}^{(k)} \setminus \{b_i^{(k)}\}, \end{cases} \quad (2)$$

where  $(\cdot \setminus \cdot)$  denotes set-difference and  $\gamma^{(k)}$  is given by

$$\gamma^{(k)} = \begin{cases} 0 & , \text{if } \exists b_r^{(k)} < 0 \text{ and } b_q^{(k)} > 0 \\ \frac{\max[\mathbf{b}^{(k)}] + \min[\mathbf{b}^{(k)}]}{2} & , \text{if } b_r^{(k)} \leq 0 \forall r, \text{ or } b_r^{(k)} \geq 0 \forall r. \end{cases} \quad (3)$$

Using this procedure, the original graph is first divided into two sub-graphs  $\mathcal{G}_1$  and  $\mathcal{G}_2$ , with corresponding connectivity matrices  $\mathbf{C}_1^{(k+1)}$  and  $\mathbf{C}_2^{(k+1)}$ , respectively given by

$$\begin{aligned} \mathbf{C}_1^{(k+1)} &= \mathbf{C}^{(k)}(\{i\}, \{i\}), \\ \mathbf{C}_2^{(k+1)} &= \mathbf{C}^{(k)}(\{j\}, \{j\}). \end{aligned} \quad (4)$$

This "splitting" mechanism is repeated over each of these connectivity matrices and over those resulting from their partition, and so on until one of the stop criteria (minimum number of nodes or minimum completeness) is met.

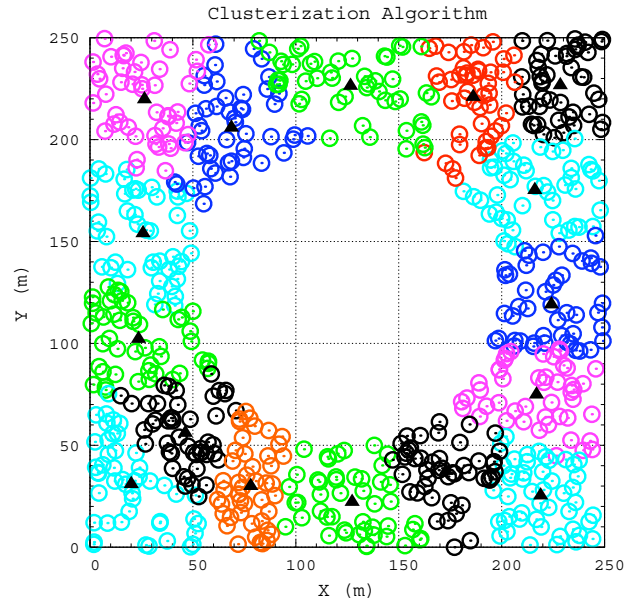


Fig. 1. Example of the output of the clusterization algorithm.

Figure 1 shows an example of a clustered network with a topological hole. The figure illustrates that the clusterization procedure effectively transforms (although not surely) "concave" topologies into unions of "convex" tiles. The clusterization degree is a configuration parameter that may vary in accordance with the network size to ensure scalability.

### B. Cluster-based Routing Protocol

The cluster-based routing protocol to be described shortly is designed under the following assumptions. During a discovery procedure (which is not discussed in this contribution), each node discovers its neighbors and forwards the connectivity information to a central processing unit, based on which the connectivity matrix of the network is built. The central unit then clusterizes the network, as described in section II-A assigns an identification (ID) number to each cluster and computes their geographical location. For short, the location of the geographical center of a cluster will hereafter be referred to as the cluster location.

Notice that a real coordinate system is not strictly necessary. In fact, methods that do not rely on actual location to route packets may be exploited to build a virtual coordinate system that can be used as input to the cluster-based routing [3], [12].

Each node is then informed of which cluster it belongs to and the IDs and geographical locations of adjacent clusters (this is a reasonable assumption since the central unit has acquired knowledge on the shortest path to all nodes after network discovery).

The routing protocol can be separated into two functional parts which are described subsequently.

1) *Inter-cluster Routing Procedure*: In addition to the cluster adjacency list (cluster IDs and locations) mentioned above, it is assumed that each node knows its own position and the position of the sinks. Actual or virtual positions can be derived distributedly during the discovery procedure [7] or computed by the central unit and passed on to the nodes [13].

When a packet is generated and has to be forwarded to a given destination (sink), the source node initially chooses, among the adjacent clusters, an intermediate destination (cluster location) which is closest to the final destination. In other words, an adjacent cluster is selected applying a greedy principle [4] based on real or virtual coordinates.

Each packet conveys, besides payload data, the source position, the sink position (final destination), the location of the next cluster (intermediate destination) and a list of the last visited clusters (the exact number of clusters that should be stored in order to avoid loops has not been optimally determined yet). Consequently, routes are established on demand, but using the clusterization information stored at the nodes, such that a packet may circumvent a hole (dead-end).

The procedure described above is referred to as inter-cluster routing.

2) *Intra-cluster Routing Procedure*: The routing procedure inside each cluster (intra-cluster routing) is largely independent of the inter-cluster procedure. In this article we considered a simple greedy forwarding mechanism for intra-cluster routing [4]. In other words, the greedy principle applied in the intra-cluster mechanism is the same as the one employed in the inter-cluster routing (with the exception that the latter includes a logic to avoid looping).

The use of (essentially) the same mechanism at inter- and intra-cluster routing is rooted on the notion that nodes can "re-use" algorithmic resources (and associated hardware/software). Furthermore, the choice of a simple greedy mechanism does not preclude the use of more sophisticated techniques, since the overall structure of the solution is essentially modular. For instance, Adaptive Load-Balanced Algorithm Rainbow (ALBA-R), which integrates Medium Access Control (MAC) and routing in its design, guarantees<sup>2</sup> packet delivery to the sink in the presence of holes (albeit at the expense of a long "transient" period) [5]. It is evident that the latter technique can be integrated into the clusterized framework discussed here as well. This is currently under consideration by the authors.

Our objective, however, is not to establish that clusterized greedy forwarding is superior to any particular alternative, but rather to show that clusterization can substantially improve the performance of routing mechanisms, even when such mechanism is as simple as greedy forwarding.

### III. PERFORMANCE ASSESSMENT

In this section, the performance of the proposed cluster-based routing protocol is assessed by means of an event-driven system-level simulator implemented using C++ Object Oriented Programming (OOP) language. The main results are presented in terms of the Cumulative Distribution Function (CDF) of the Packet Delivery Success Ratio (PDSR) as a function of the corresponding Average Packet Delivery Latency (APDL), and in terms of the distribution of route path length (in number of hops). A few routing trajectories obtained during systemic simulations are also shown for illustration.

<sup>2</sup>Guaranteeing packet delivery may not always be desirable. In some cases, not all generated traffic is desirable traffic (think of environmental sensing with sample redundancy). In others, even if desirable, the costs incurred by a 100% packet delivery may be too large, imposing a compromise. The latter case is perhaps closer to the examples explored hereby.

#### A. Simulation Environment

A dynamic radio network simulator is used to evaluate the overall network performance and effectiveness of the proposed routing protocol. All simulations were performed using topologies generated randomly, with nodes uniformly distributed over a 250m-wide square with a concave hole of shape resembling "Pacman" placed in the middle.

The minimum node degree and corresponding radio range are set so as to guarantee the connectivity of the network [14], considering a connectivity probability of at least 99%.

Two sets of simulations were run, one with all nodes awake fulltime (duty-cycle of 100%) and another with a nodal duty-cycle of 10%. All nodes operate independently and traffic is Poisson-distributed, with adjustable packet inter-arrival time  $\lambda = 0.5\text{pkt/s}$ , assigned randomly to the nodes.

The nodes' buffers are considered long enough to avoid packets to be discarded due to overflow. Consequently, nodes can serve as relays for several packets simultaneously, although a node can only transmit or receive one packet at a time (half-duplex). When a node has multiple packets to relay, the packets are queued in the buffer and drawn on a first-in-first-out basis, with buffer rotation applied whenever a relay cannot be found for a certain packet (no priorities). The buffer rotates all its packets until transmission to the next relay is successful. The time a packet stays queued in a node's packet buffer, including the time the node sleeps, is added to the final delivery latency metric.

Detailed radio propagation models are not considered in this work. Instead a simplified two-state (busy, idle) channel model is used to emulate the air interface occupancy during packet transmissions. Nodes only transmit when the channel is idle.

No explicit contention-resolution mechanism [15], [16] was used since the objective herein is to evaluate the routing technique only. While this does not affect the results in general, it is reasonable to assume that if the time spend in contention resolution were to be added to the latency metrics observed in the simulations, the small increase in latency seen with the proposed clusterized routing protocol under a nodal duty-cycle of 100% (see figure 4) would no longer be visible.

It is assumed in the simulation set-up that network discovery had been already performed and that the network is in a steady-state regime of operation, when the routing protocol itself starts to operate. An underlying assumption is that the network discovery was coordinated (or at least overlooked) by the central unit which, in the process, acquired the entire network connectivity matrix.

TABLE I  
SIMULATION PARAMETERS.

Parameter	Value
Area	Square (250 m)
Radio range	24.77 m
Duty Cycle	100% and 10%
Backoff Interval	0.219s
Awake Interval	0.016s
Transmission Time Interval	0.0521s
Cluster Size	25, 35 Nodes
Number of Nodes	500 Nodes
Packet inter-arrival rate	0.5 packets/s

At the beginning of each simulation, a sink is chosen amongst the nodes around the coordinates  $X = 250\text{m}$ ,  $Y = 125\text{m}$ , such that it is diametrically opposite to the concave portion of the hole (see figures 2 and 3). This configuration accounts for a worst-case scenario, in the sense that the number of nodes subject to dead-end problems is maximized.

Key simulation parameters are listed in Table I. Some of the figures were taken from [17].

### B. Simulation Results

In this section, the proposed cluster-based routing protocol is evaluated addressing both its applicability in WSNs and its capability of dealing with dead-ends. The network performance is evaluated in terms of PDSR, APDL and the distribution (histogram) of the number of hops (in excess of the shortest path) required by a packet to travel between its source and its final destination.

In order to ensure the statistical significance of the results, each simulation lasts the time necessary for the sink to collect 5000 packets.

Figure 2 depicts a few exemplary routing trajectories obtained when sleeping interval is disabled (duty cycle equal to 100%). Notice that the simulation scenario presents a harsh network topology. It is remarkable that even in this extreme configuration nodes located at the concave portion of the hole (Pacman's mouth) are still able to propagate their packets towards the sink, albeit at the expense of a higher APDL, due to the longer trajectories circumscribing the hole.

Similarly, figure 3 shows packet trajectories towards the sink when the nodal duty-cycle is 10%. As can be seen from this figure, the trajectories spread slightly, since packets now may deviate from the shortest path more frequently as result of relays being chosen among nodes that are awake.

Together, figures 2 and 3 illustrate the remarkable effectiveness of the proposed technique, demonstrating its ability to find routes even through the narrow corridors and in the presence of such a prominent concave hole, although the underlying relay mechanism is just the simple greedy forwarding algorithm.

Figure 4 shows the PDSR as a function of the APDL, for different levels of network clusterization with all nodes awake at all times. It is found that the simple greedy forwarding is severely affected by the dead-end and fails to route packets originated from the concave cone region towards the sink. This explains why the PDSR of the greedy algorithm is around 50%, *i.e.*, only the traffic generated on the convex side of the hole is successfully routed.

Conversely, the proposed clusterized routing solution is capable of delivering a substantially larger fraction of the traffic to the sink. It is also found that the cluster size has an impact on the effectiveness of the method. For instance, when the clusterization is such that clusters have at least ( $N = 25$ ) nodes, the PDSR is higher.

Such a higher delivery ratio, however, comes at the expense of extra latency, since packets tend to circulate inside the concave region of the hole (Pacman's mouth) for longer, before finally being routed around it.

Notice that very small clusters may also degrade PDSR, because this may decrease the probability of a node to find a relay in a selected adjacent cluster (intermediate destination).

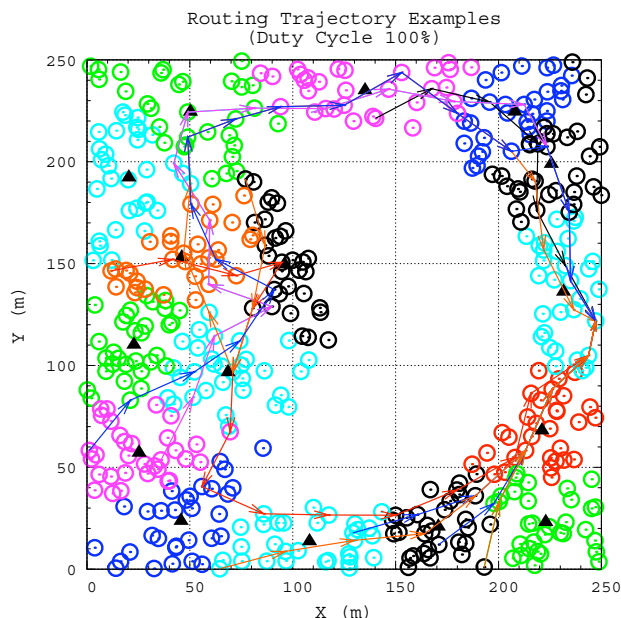


Fig. 2. Illustrative trajectories using cluster-based routing without sleeping interval (average cluster size equal to 25 nodes).

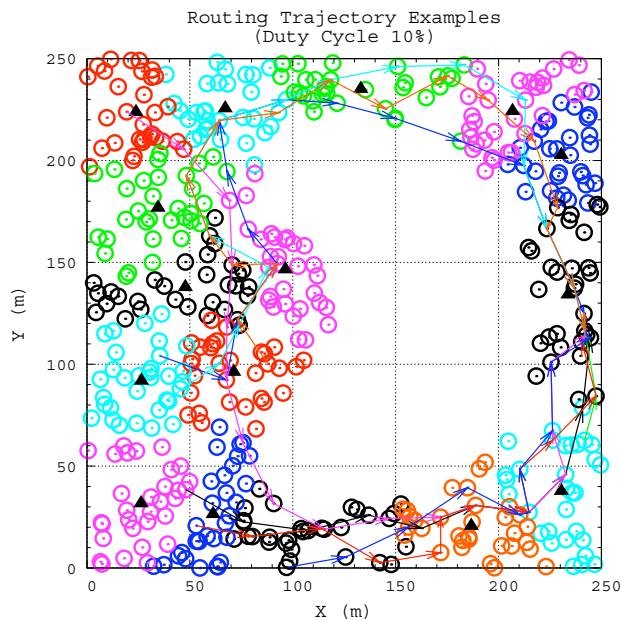


Fig. 3. Illustrative trajectories using cluster-based routing with sleeping interval (average cluster size equal to 25 nodes).

Just as well, PDSR may also decrease if clusters are too large since this increases the likelihood that it contains a dead-end itself, although this is less likely due to the tendency of the clusterization algorithm to group nodes according to their connectivity. While it is true that some optimization may be achieved by changing cluster sizes, it is clear that the issue is less critical than the selection of adequate landmarks.

Next, figure 5 shows the PDSR as a function of the APDL, for different levels of network clusterization with a duty cycle of 10%. Notice that the network topology is not static anymore, since now nodes alternate between awake and asleep states following their own duty cycle (even though there is no mobility, the network topology now varies with time).

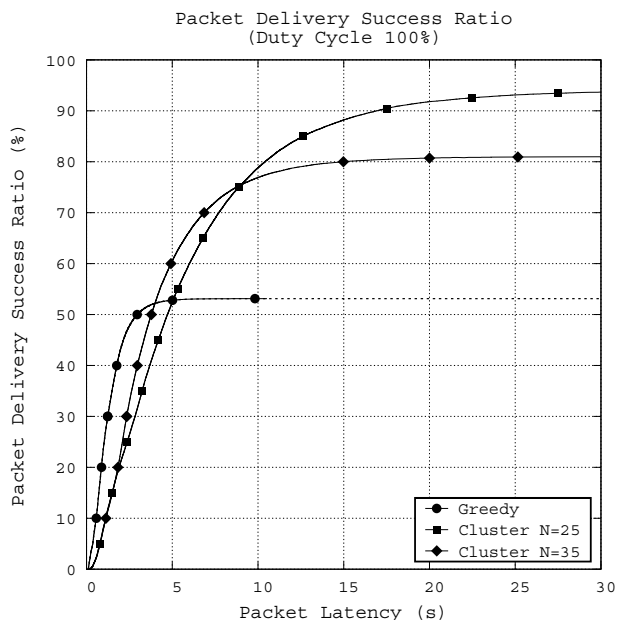


Fig. 4. Packet delivery success ratio with duty cycle of 100 %.

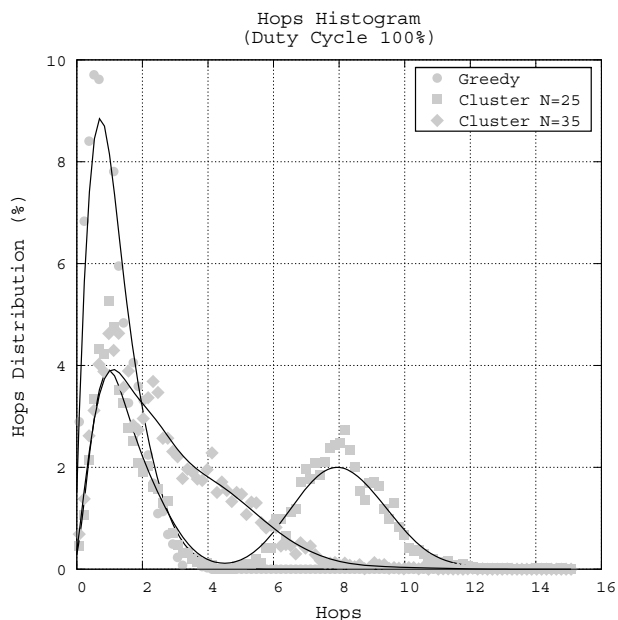


Fig. 6. Distribution of excess-hop with duty cycle of 100 %.

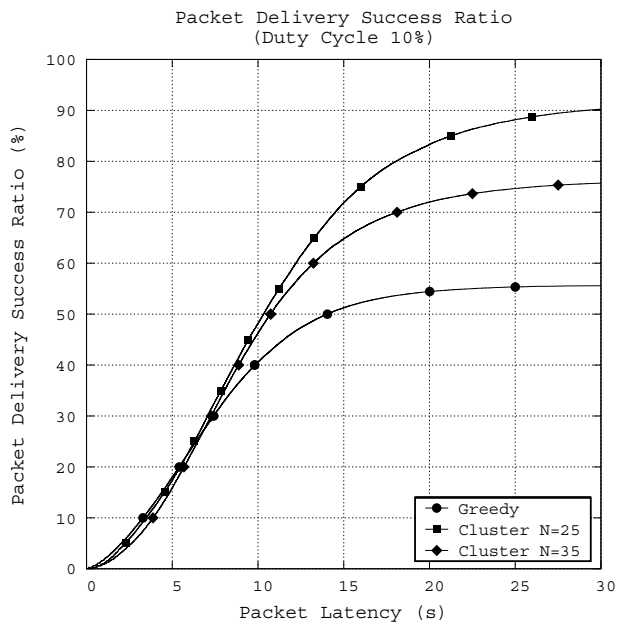


Fig. 5. Packet delivery success ratio with duty cycle of 10 %.

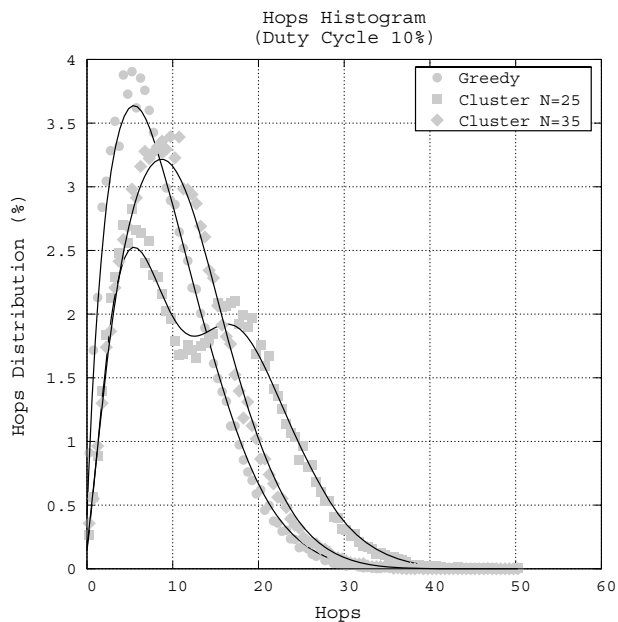


Fig. 7. Distribution of excess-hop with duty cycle of 10 %.

Comparing these results with those of figure 4, it is seen that APDL, now dominated by the time packets spend in the buffer of nodes awaiting a relay to be selected within the greedy forward procedure, is increased overall. Consequently, no significant difference in APDL is observed amongst all different systems. In turn, the proposed routing technique proves to be resilient enough to absorb the variable network topology and to continue delivering a much higher PDSR.

Finally, figures 6 and 7 show the distributions of the number of hops – in excess of what would take if packets were to be forwarded in a straight-line, with zero awaiting time (perfect relaying) – required by the routing protocols under the simulated conditions.

The figure clarifies why the APDL of the proposed method is larger than that of the greedy algorithm, as per figure 4. The second mode of the distribution corresponding to the proposed technique with  $N = 25$  shown in figure 6 illustrates how the proposed routing algorithm allows more nodes – namely, those at the concave portion of the hole – a path to the sink.

Furthermore, as shown by the median of all distributions seen in figure 7, the number hops in excess of straight-line-hopping in the case of a network with small nodal duty-cycle increases, regardless of the routing method utilized. This, again, explains why the increased delayed penalty associated with the proposed solution as implied by figure 4 no longer exists when the network is not static.

## IV. CONCLUSION

We proposed a cross-layer algorithm for geographic routing in WSNs, which is robust to dead-ends and is resilient to topological variations due to network dynamics. The solution combines ideas of network tessellation (clusterization) with greedy forwarding, without suffering from the problems afflicting landmark-based alternatives.

Instead of starting from pre-determined landmarks and tessellating the network around them [7], [9], as in conventional landmark-based techniques, we first tessellate the network based on connectivity information – which amounts to a clusterization – and then define virtual landmarks to identify them. The algorithm employed to clusterize the network is based on a recently-discovered graph-spectral property that captures the connectivity among nodes [11]. In addition to not requiring landmarks to be known *a priori*, a major advantage of this "reversed" tessellation approach is that the clusters are (by construction, as a result of the aforementioned clusterization technique) composed by groups of nodes that are highly (mostly) interconnected, within which "greedy forwarding methods based on local coordinates are likely to work well" [7, pp. 340].

It could be said that unlike earlier landmark-based approaches, where well-placed landmarks are required in order to construct good clusters, in our approach good (virtual) landmark locations are "discovered" to represent well-constructed clusters. The new twist in the method is that, while there is no real (general) way to know good locations for landmarks, connectivity information (which may be obtained via an appropriate discovery procedure) proves sufficient to construct good clusters.

It was shown that cluster sizes may affect the resulting PDSR to APDL. In other words, cluster sizes can be varied so as to allow for different trade-offs between PDSR to APDL to be reached. It was also found, however, via extensive simulations, that the dependence of the overall technique on the cluster sizes is not as strong as the dependence of landmark-based routing mechanisms on the location of selected landmarks. Alternatively, we may say that while there is room for further improvement via cluster size optimization<sup>3</sup>, the proposed technique is less sensitive to a bad choice of cluster size than landmark methods are on the location of landmarks.

All in all, our extensive simulations have confirmed that the technique can substantially improve the PDSR in networks with large concave holes, with no or little impact on APDL, even if the simple greedy forwarding mechanisms is employed. There is, however, no reason not to employ more sophisticated mechanisms combined with the clusterization idea. Such possibilities are currently being considered by the authors.

## ACKNOWLEDGMENT

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<sup>3</sup>In addition to cluster size optimization another possibility is to improve the clusterization by taking into account nodal duty-cycle. This is an interesting problem that may benefit from recent progress on mathematical models for dynamic networks *e.g.* [18].

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