Analysis of Contention-based Relay Selection Mechanisms in Autonomous Multi-hop Networks

Carlos H. M. de Lima and Giuseppe Thadeu Freitas de Abreu
Centre for Wireless Communications
University of Oulu
P. O. Box 4500FIN-90014
Email: [carlos.lima, giuseppe.abreu]@ee.oulu.fi

Abstract— **A fundamental question of interest in multi-hop networks is whether routing should be done in a smaller number of longer hops, or larger number of shorter hops. However, underneath these aspects of the routing problem is the *number* of neighboring nodes involved in the relay selection taking place at each hop so as to realize whichever hopping strategy adopted. A larger number of nodes increases the likelihood of finding an adequate relay, but increases selection overhead, and vice-versa. In this article, we investigate the *expected* cost (in time consumed) and effectiveness (in spatial adequacy to the hopping policy) incurred in relay selection processes employing Contention-based Geographic Forwarding (CGF) methods, as a function of the number of nodes involved. In particular, Probability Generating Functions (PGFs) are utilized to compute the distribution of Contention Resolution Intervals (CRIs) and quantify overhead, while stochastic geometry is used to model topological aspects of the network and quantify expected progress. Both a random and an optimized (auction-based) Relay Selection Algorithms (RSAs) are studied and compared, with the advantages of the auction-based approach established analytically. From an overhead point of view, the selection of furthest and nearest relays are equivalent, but our results indicate that the long-hop routing approach is less sensitive to the number of contending nodes than the short-hop alternative.**

I. INTRODUCTION

In wireless ad hoc and sensor networks, multi-hop links need be established when sources communicate with destinations farther than their transmission range. In most applications, network and channel dynamics require that routes be re-established frequently, with each hopping step involving the selection of a relay that satisfies a criterion defined by the routing strategy [1]. Such a dynamic relay selection procedure is performed with the help of contention-based Random Multiple-Access (RMA) methods [2], where packet collisions are either avoided or resolved autonomously. In the Contention-based Geographic Forwarding (CGF) context improvements can be made if relay nodes utilize knowledge of their own location as side-information [3].

A fundamental question regarding the dynamic formation of multi-hop links is whether routing should be done in a smaller number of longer hops, or in a larger number of shorter hops. There is an on-going debate in the literature over which of these strategies is the most adequate [1], but a clear answer has not emerged. What is generally true is that the number of hops impacts on the time required for packets to reach their destination, while the hop length impacts on the reliability of the communication channel at each hop, with the overall network performance depending on a balance between these two factors.

Whichever the hopping strategy, however, it is by means of a Relay Selection Algorithm (RSA) that the source and potential relays undergo multiple rounds of packet exchange attempts until the source is capable of receiving a collision-free packet from a relay that meets the criterion. The efficacy of RSAs will therefore also play a major role in the rates ultimately achieved over multi-hop links, and thus on the overall capacity ad hoc networks. Unfortunately, proportionally little effort has been made to properly characterize the cost/effectiveness of RSAs in multi-hop networks, which can be measured by the average time, or more accurately, the number of Contention Resolution Intervals (CRIs) required for the selection to be completed, and by the expected adequacy of the selection produced, respectively.

In this article, we investigate the expected cost and efficacy of relay selection processes employing CGF methods, as a function of the number of nodes involved. The first of these metrics is primarily determined by the number of neighbors[1] within the transmission range of the source that actually participate, while the second is related to the hopping policy and topological features of the network. We quantify analytically the protocol overhead and the expected progress of the Medium Access Control (MAC) relay selection mechanism employing both a Standard Tree Algorithm (STA) and the auction-based schemes proposed in [3]. In particular, Probability Generating Functions (PGFs) are utilized to compute the distribution of CRIs and quantify overhead, while stochastic geometry is used to model topological aspects of the network and quantify expected progress.

The delay distributions are characterized by using the traditional approach of conditioning on the initial multiplicity of the conflict [4]. Simulation results are also provided which corroborate the theoretical analysis [3], [5].

One outcome of the analysis that relates to the aforementioned debate over short-hop versus long-hop routing is that furthest neighbor relay selection is less sensitive to the number of contending nodes than the nearest neighbor alternative. This suggests that relay selection overhead can be made more efficient – *e.g.*, by reducing the eagerness with which nodes make themselves available to relay packets (cooperation factor) – under a long-hop routing policy with less impact onto the strategy than it can under a short-hop routing approach.

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[1] In average terms, this number can be seen as a design parameter. In sensor networks it can be determined by deployment (density) and in ad hoc networks it can be controlled by the transmission range or a “cooperation factor”.

The remainder of the article is organized as follows. The formulation of the CRI distributions incurred by the RSA are discussed in section II. The numerical procedure to invert the PGFs and recover the corresponding Probability Mass Functions (PMFs) is presented in section III. In section IV, the expected progress that is achieved at each hop by using the CGF schemes is characterized. Section V provides a discussion on the applicability of our results in wireless multi-hop networks. Final remarks and perspectives for further investigations are given in section VI.

II. Contention-based RSAs

The analysis is driven considering two distinct contention-based Relay Selection (RS) mechanisms: (i) a purely random solution based solely on the splitting tree algorithm for performing RMA communications [4]; and (ii) a game-theoretical approach exploiting location information to avoid collisions and, whether necessary, to improve the contention resolution period [3].

The Contention Resolution Algorithms (CRAs) are characterized in terms of the corresponding CRIs necessary to select the most suitable relay – in terms of geographic advancement towards the final destination – among the eligible neighbors. In order to characterize the incurred delay of such solutions so as to forward packets towards destinations, the PGF is initially used to represent the PMF of the CRI by means of power series, and therewith the entire distribution is recovered by numerically inverting the corresponding PGF using the Fourier series method introduced in [5].

A. Delay analysis of the STA-based RSA

The STA scheme is based on the splitting tree algorithm for RMA communications [4]. The CRA is implemented considering the obvious Blocked Access Protocol (BAP) which means that no relay is allowed in the transaction once the contention has been initiated. The performance of the STA-based RSA is addressed by means of computational simulations in [3].

Succinctly, the source node initiate the relay selection transaction by issuing a Request to Send (RTS) packet. Afterwards, neighbor nodes that had listened to the source's requisition split themselves randomly and independently based solely on the common probability that dictates the likelihood of accessing the shared channel.

Whether the eligible relays collide, nodes that have transmitted in the previous slot decide to retransmit or to refrain tossing a Q-sided coin with fair probabilities. Herein, binary trees are actually used in the investigations, though the observations are equally applicable to higher number of splitting groups. And yet the source node should receive the replies from all the candidate relays so as to select the next relay greedily – to select the closest node to the destination whether there is one available.

1) PGF of the CRI Length: In this section we derive the PGF of L_N – the conditional CRI length when N nodes initially collide. Equation (1) generalizes the conditional CRI length considering a Q-sided fair coin.

$$L_N = \begin{cases} 1 & \text{, if } N = 0, 1, \\ 1 + \sum\limits_{j=1}^{Q} L_{I_j} & \text{, if } N \geq 2. \end{cases} \tag{1}$$

where I_j is the discrete random number of candidate relays that tossed the j value on the Q-side coin.

The PGF of the L_N (CRI length) is defined as follows,

$$G_N(z) \triangleq \sum_{k=0}^{\infty} P\left\{L_N = k\right\} z^k = E\left\{z^{L_N}\right\}. \tag{2}$$

where L_N is a discrete Random Variable (RV) assuming non-negative integer values, and $E\left\{\cdot\right\}$ is the expectation value.

And so taking the conditional expectation on the right-hand side of (2) leads to

$$E\left\{z^{L_N}\right\} = E\left\{E\left\{z^{L} | I_1, \ldots, I_Q\right\} | N\right\}, \tag{3}$$

$$G_N(z) = z \sum_{i_1,\ldots,i_Q}^{N} \binom{N}{i_1, \ldots, i_Q} \prod_{j=1}^{Q} Q_{L_{I_1}}(z) {P_j}^{i_j}. \tag{4}$$

where ${P_j}^{i_j}$ is the probability of i_j nodes flip the j side of the Q-sided fair coin. The summation iterates over all possible combinations of the splitting groups $i_1, \cdots, i_Q$.

Hereafter we restrict our analysis to the binary case ($Q = 2$). Whenever a collision occurs, *i.e.*, $N \geq 2$, the candidate relays split themselves into two subsets. Each sub-CRI is statically indistinguishable from a CRI initiated by the same number of contenders.

$$I_1 = i, I_2 = N - i. \tag{5}$$

Equation (6) yields the probability of exactly i nodes toss 0 (first splitting group), and then transmit in the very next frame,

$$B_{N,i} = \binom{N}{i} p^i (1-p)^{n-i}. \tag{6}$$

where p corresponds to the probability of tossing 0 when using the fair binary coin.

Finally, the PGF of the STA-based strategy is then given by equation (7),

$$G_N(z) = z \sum_{i=0}^{N} B_{N,i} G_i(z) G_{N-i}(z). \tag{7}$$

where $G_i(z)$ addresses the collision among i nodes that flipped 0 (first subset), and $G_{N-i}(z)$ corresponds to the additional slots to resolve the collision among $N - i$ nodes that flipped 1 (second subset).

B. Delay analysis of the Auction-based RSA

The auction-based RSA deals with the collision avoidance/resolution stages by employing economic game-theoretical concepts. Typically, when a source node has a packet pending for transmission, an RTS/ CTS handshake is triggered to avoid simultaneous transmissions [2]. However, each suitable relay dwelling on the source node's radio range, which have received a RTS packet, will reply with a Clear To Send (CTS) packet and collisions may occur. Thus, an appropriate mechanism has to be employed in order to cope with the imminent contention.

In [3] Dutch auctions are proposed as an effective alternative to address the RS process in conjunction with a Random Channel Access (RCA) solutions. In such transactions, source node plays the role of an "auctioneer" and potential relays are the "bidders". Indeed, Dutch auctions are extremely convenient to sell goods – assignment of network resources – quickly.

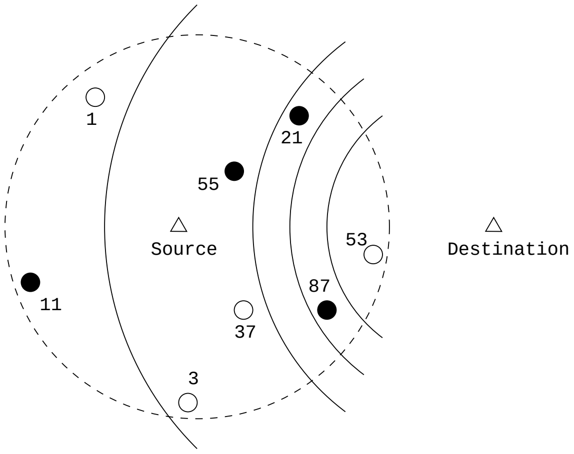

Fig. 1. Illustration of the auction-based RSA. Definition of forwarding regions. Relay candidates are identified by filled-circles. The dashed circle delimits the source's radio range.

The reasons are two-fold, the auction ends with the very first bid and the auctioneer may set accordingly the decreasing rate of artifact value (depreciation rate) aiming at quickening the auction [6].

Figure 1 shows the computation of the forwarding regions. For the first auction round, the relay candidates are divided into three groups, $\{[21,87],55,11\}$. Since nodes $\{[21,87]\}$ reply at the same time slot, a collision occurs. The source node then detects the collision in the first slot and recompute the forwarding regions accordingly. When the BAP is considered, the contending nodes can also recompute the forwarding regions independently. Nodes $\{55,11\}$ also detect the collision and, as nodes of higher priority have already replied, drop out. Afterwards, potential relays in the first colliding area are reordered in the sequence $\{[],87,21\}$. Finally, after the first idle slot, node 87 replies and the auction stops.

1) PGF of the CRI Length: In accordance with the auction-based RSA, potential relays that have not replied in the previous slot but detected a collision just drop out of the ongoing transaction. Indeed, the "tree pruning" means that whenever the first subset visited after a collision leads to another collision, the second subset is dropped. Therefore, for $i>1$, instead of the CRI having length $B_{N,i}G_i(z)G_{N-i}(z)$, the tree pruning procedure leads to a shorter length $B_{N,i}G_i(z)$. Equation (8) yields the PGF of the auction-based RSA [4].

$$G_N(z)=z^2B_{N,0}G_N(z)+z^2B_{N,1}+z\sum_{i=2}^{N}B_{N,i}G_i(z). \quad (8)$$

where the first term accounts for case when no reply is issued in the first slot, and the second term addresses the cases when there is only one eligible neighbor in the first decision region.

III. NUMERICAL INVERSION OF PGFS

The PMF of the CRI length is recovered from the corresponding PGF by means of a numerical inversion technique introduced in [5] that relays on the Fourier series method. The CRI distribution is approximated by equation (9),

$$\tilde{P}\left\{L_N=k\right\}=\frac{1}{2kr^k}\sum_{j=1}^{2k}(-1)^jRe\left[G_N\left(re^{\frac{\pi ji}{k}}\right)\right], \quad (9)$$

Additionally, a predetermined error bound is derived for $0<r<1$ and $k\geq 1$ as follows,

$$\left|\tilde{P}\left\{L_N=k\right\}-P\left\{L_N=k\right\}\right|=\frac{r^{2k}}{1-r^{2k}}. \quad (10)$$

IV. AUTONOMOUS POSITION-CENTRIC NETWORK ROUTING

In this section, the influence of the hop-basis RS procedure on the End-to-End (E2E) delivery latency when transmitting packets through multi-hop links is addressed. In order to perform such investigations, two distinct contention-based Geographic Forwarding (GF) strategies are considered: sectoral and circular lens forwarding regions (see figure 1) [7]. Stochastic geometry is used to model the network deployment as a Spatial Poisson Process (SPP). The distribution of the distances of the n-th nearest neighbor for the geographic forwarding decision regions considered herein are the generated [8], [9].

A. System Model

Nodes are uniformly and randomly distributed over the network area. In fact, it is assumed that the distribution of distances among nodes follow the homogeneous Poisson Point Process (PPP). Indeed, it is assumed that the conditional distribution of the number of neighbors within source's radio range is a general 2-dimensional Binomial Point Process (BPP). A simplified connectivity model based on the Unit Disk (UD) is employed – protocol model. Therefore, awake neighbor nodes within sources' radio range are considered eligible relays. It is assumed that distinct routing trajectories may be established between source and final destination as consequence of network dynamics. The packet buffer is assumed to have unitary size.

B. PDF of the distance of the n-th nearest neighbor

The Complementary Cumulative Distribution Function (CCDF) of the n-th nearest neighbor $\bar{F}_{R_n}(r)$ is the probability of existing less than n points in the decision region $\mathcal{R}$ [8].

$$\bar{F}_{R_n}(r)=\sum_{0}^{n-1}\binom{N}{k}p^k(1-p)^{N-k}. \quad (11)$$

where $p=\dfrac{\lambda_2(B)}{\lambda_2(W)}$, $W\subset\mathbb{R}^2$, $B\subseteq W$, and $\lambda_2(\cdot)$ corresponds to the Lebesgue measure in the plane [9].

Whenever a and b are integer values, the Regularized Incomplete Beta Function (RIBF) can be expressed as in equation (12),

$$I_x(a,b)=\sum_{k=a}^{a+b-1}\frac{(a+b-1)!}{k!(a+b-1-k)!}x^k(1-x)^{a+b-1-k}. \quad (12)$$

And yet the CCDF can be rewritten in terms of the RIBF as follows,

$$\bar{F}_{R_n}(r)=I_{1-p}(N-n+1,n). \quad (13)$$

The Probability Distribution Function (PDF) of the n-th nearest neighbor is then derived as,

$$f_{R_n}(r)=\frac{\partial}{\partial r}I_p(n,N-n+1). \quad (14)$$

where $I_x(a,b)=1-I_{1-x}(b,a)$.

V. RESULTS

In this section the CRI is analyzed for distinct RSAs. Moreover, the expected advancement provided by the distinct CGF schemes are also evaluated relative to the corresponding resolution delay incurred by the conflict resolution algorithms.

Figure 2 presents the PMF of the CRIs that is achieved when the STA-based Contention Resolution (CR) mechanism is employed. The distribution of the CRI is generated for increasing values of contending relays. computational simulations are also provided so as to corroborate the theoretical analysis using the PGFs. The network simulator introduced in [10] is used to perform the simulation campaign. When using the purely random CR approach the CRI significantly lengthens by considering increasing number of contending relays. In fact, the resolution of the conflict may linger too much time before the contention is resolved and the next-hop relay is elected.

For the Auction-based RSA, the PMF of the CRIs is presented in figure 3. The impact of the initial number of colliding nodes on the duration of the contention resolution period is still observed, though in much lesser extent. As can be seen from the figure, by autonomously using the location information eligible nodes can independently split themselves into priority groups and effectively quicken the RS transactions. It is clear that location awareness significantly improve the CR capability of the game-theoretical RS process. By comparing figures 2 and 3 and considering the curves of two eligible relays, the probability of resolving the contention in seven slots is nearly 12% for the STA-based procedure, whereas the same CRI has probability of approximately 7% for the auction-based RS strategy.

Figure 4 shows the expected value of the n-th neighbor distance for the CGF schemes defined in section IV, namely sectoral and circular lens decision regions.

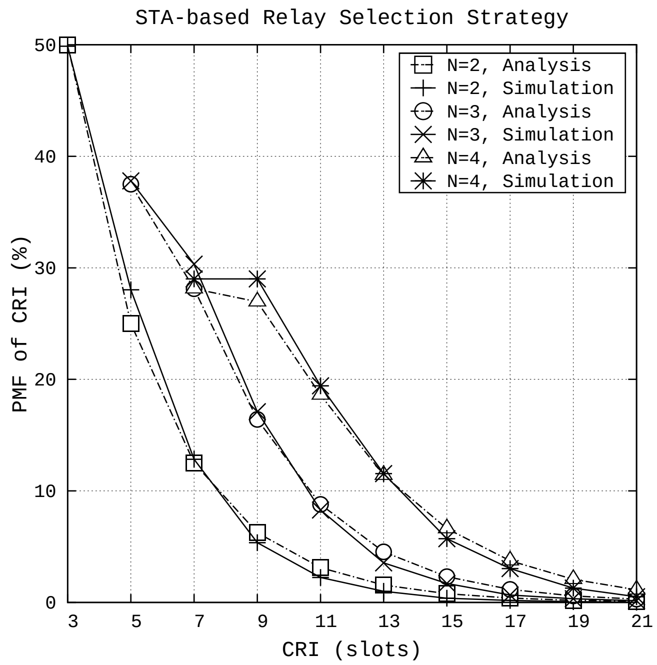

Fig. 2. PMF of the CRI length for the STA-based RSA. The Number of relays identify the number of candidate nodes that initially collide.

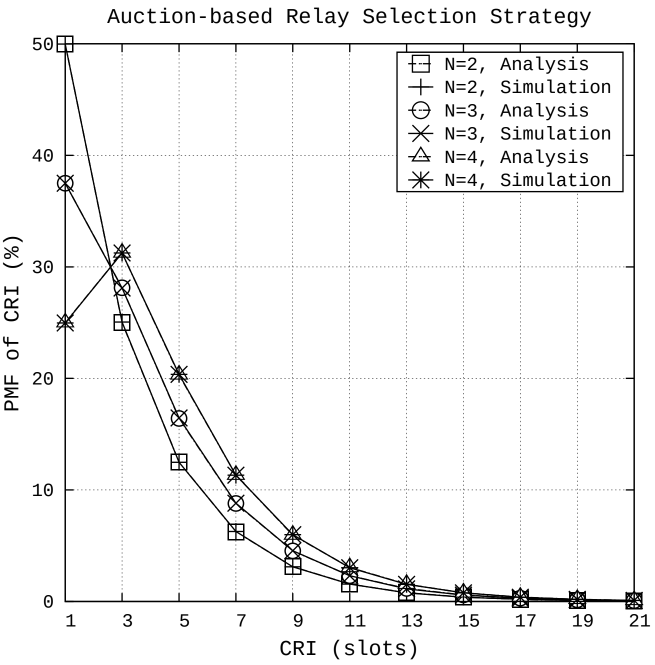

Fig. 3. PMF of the CRI for the auction-based RSA. The Number of relays identify the number of candidate nodes that initially collide.

The higher the number of neighbor nodes within source's transmission range, the further is the expected distance of the furthest eligible relay. However, for the evaluated number of relays (from 2 up to 11 nodes), the distance of the furthest nodes do not increase substantially by considering higher number of relay candidates. Conversely, the expected distance of the nearest neighbor not only experience higher variance in hop length, but also the expected advancement become even smaller since the nearest node is found closer to the source node when the number of candidate relays increases.

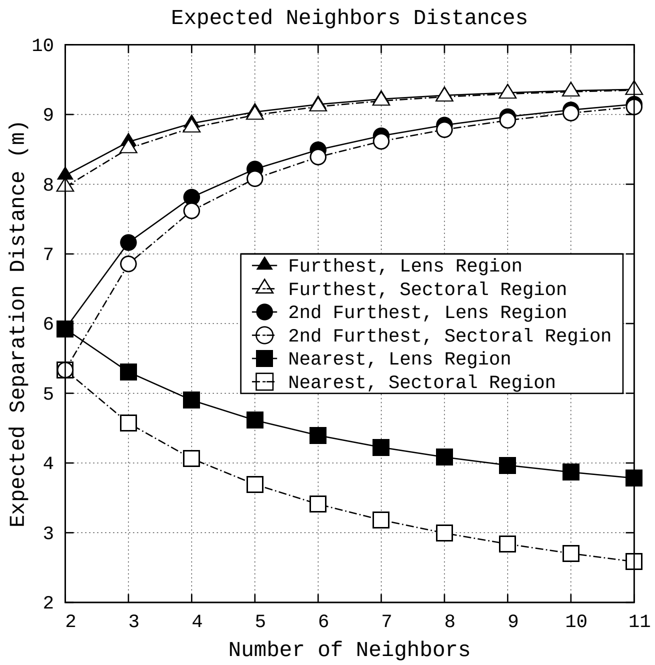

Fig. 4. Expected separation distance of the n-th nearest neighbor for the CGF schemes (radio range of 10m).

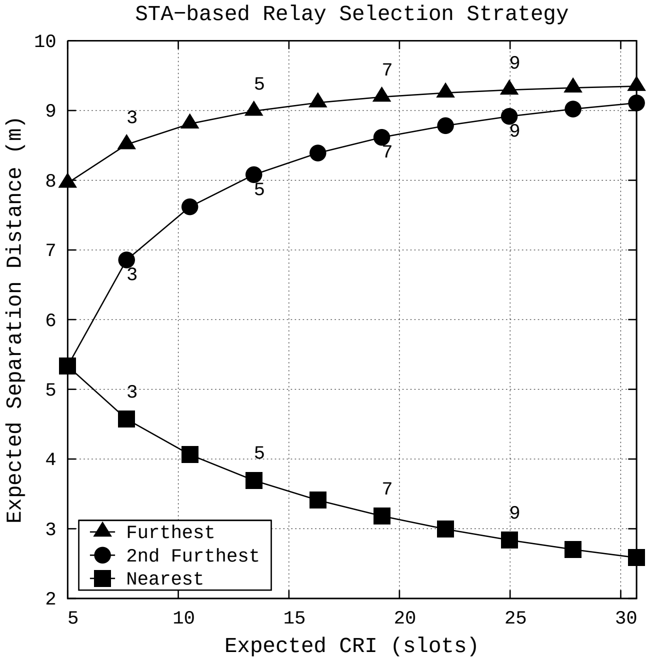

Fig. 5. Expected distance to the n-th nearest neighbor related to the expected value of the CRI for increasing number of contending relays.

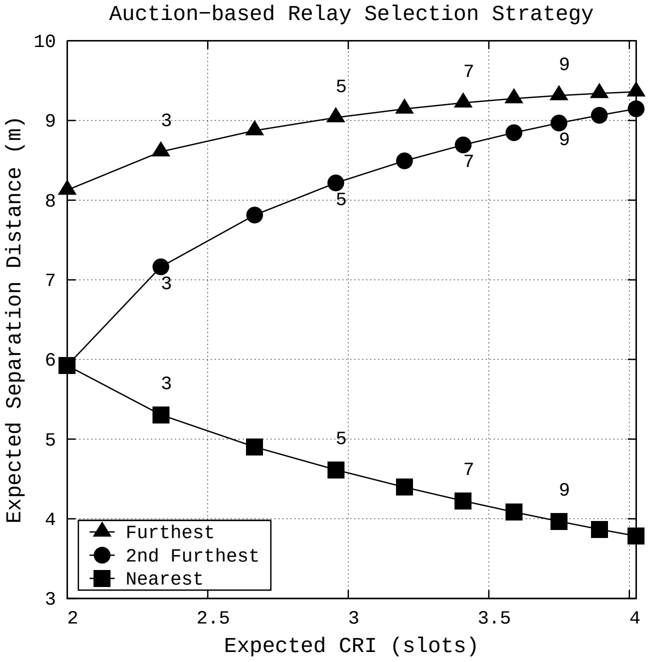

Fig. 6. Expected distance to the n-th nearest neighbor related to the expected valure of the CRI for increasing number of contending relays.

Figures 5 and 6 relate the expected distances of eligible relays to the corresponding CRIs. As can be seen from the foregoing figures, the CRI militates against the benefit of having higher number on eligible relays withing radio range, since depending on the initial number of colliding nodes the contention resolution may linger too long and then compromises the overall performance. While the two decision regions provide comparable advancements, except for the nearest neighbor case, the CRIs undergone by using distinct RSA is the determinate factor for system performance. Therefore, it is advantageous to keep small the number of relays that entangle into the election process, since the relay selection procedure adds substantially to the packet propagation delay in a hop-basis.

VI. FINAL REMARKS AND PERSPECTIVES

In this contribution the cost of selecting the next-hop relay when employing CGF methods is investigated. Specifically, the CRI required to find a suitable relay in multi-hop scenarios is characterized. Stochastic geometry is used to appropriately model the topology of such communications networks, while PGFs are utilized to recover the distribution of the CRI incurred by the assessed conflict resolution procedures.

The performance of the STA-based approach is compared against the proposed game-theoretical strategy which exploits location as side-information to quicken the CRIs. The auction-based approach remarkably outperforms the STA-based solution. Indeed, game-theoretical RSA substantially reduces the protocol overhead of establishing active connections in autonomous multi-hop networks allowing more efficient reuse of the shared communication channel. Moreover, regarding the network and channel dynamics, the RS procedure carried out at each hop may become a dominant aspect for the overall performance, mainly if the (re)establishments of such multi-hop links need to be performed frequently.

It has been shown that the time necessary to select the next-hop relay in multi-hop relay networks may become determinant to the overall system capacity. In fact, it is for further investigation the quantification of the achievable transmission capacity of such relaying networks taking also into account the cost of the MAC protocol overhead [11].

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