This paper studies the effect of coding on the energy consumption of multihop communication in Wireless Sensor Networks (WSN). In WSNs, there are strict energy consumption demands because the battery energy is limited and the lifetime of the network must be maximized. We have used an analytical model for the radio energy consumption to study how different codes can affect the energy consumption. The tradeoff between coding overhead and energy consumption per information bit with different bit error probabilities on the channel and different number of hops is presented in our numerical results. These results show that the energy efficiencies of the codes used in a multihop routing strategy are strongly dependent on the channel conditions and the number of hops used.

I. INTRODUCTION

Recent developments in wireless communication and electronics have made possible the construction of low-cost Wireless Sensor Networks (WSN). In recent years, a lot of research has been done in the area of sensor networking to evaluate not only low-cost but also low-power WSNs. With current technology, the wireless communication between sensor nodes is the greatest energy consumer and the effective way to decrease energy consumption in sensor nodes is to optimize energy efficiency of the communication. In this paper, we are studying how different codes affect the energy consumption of multihop communications. Coding and multihop effects on energy consumption have been studied before but we combine these two things to make results more accurate. The energy consumption of multihop communications with a different number of hops between the original information source and the sink is studied when various common channel codes are used to make transmissions more reliable. In our calculations we have assumed a slowly Rayleigh fading channel and a linear multihop model which also assumes that the distances between nodes are equal.

II. RADIO MODEL

Transmission and reception tasks are the largest consumers of energy in wireless embedded networks. We use a radio model [1] to represent and analyze this energy consumption in a multihop environment. Fig. 1 illustrates the energy $E_L(m,d)$ spent when sending $m$ bits over a wireless hop link of distance $d$.

$$E_L(m,d) = E_T(m,d) + E_R(m)$$

where $E_T$ is the energy spent at the transmitter and $E_R$ the energy spent at the receiver. The transmitter energy consumption can be expressed as:

$$E_T(m,d) = E_{TC}(m) + E_{TA}(m,d)$$

where $E_{TC}$ is the energy used by the transmitter circuitry and $E_{TA}$ is the energy required by the transmitter amplifier to achieve an acceptable signal to noise ratio at the receiver. Assuming a linear relationship for the energy spent per bit by the transmitter and receiver circuitry Equation 2 can be further simplified as:

$$E_T(m,d) = m(e_{TC} + e_{TA}d^\alpha)$$

$$E_R(k) = me_{RC}$$

where $e_{TC}$, $e_{RC}$ and $e_{TA}$ are hardware dependent constants.

An explicit expression for $e_{TA}$ can be derived as [2]:

$$e_{TA} = \left( \frac{S}{N} \right) \frac{(NF_{Rx})(N_0)(BW)}{L_0} \frac{1}{d_0^\alpha}$$

where $(S/N)_r$ is the desired signal to noise ratio at the receiver’s demodulator, $NF_{Rx}$ is the receiver noise figure, $N_0$ is the thermal noise floor in a 1 Hz bandwidth, $BW$ is the channel noise bandwidth, $L_0$ is the path loss attenuation at $d_0$ meters, $\alpha$ is the path loss exponent, $G_{ant}$ is the antenna gain, $\eta_{amp}$ is the transmitter power efficiency and $R_{bit}$ is the raw bit rate in bits per second. This expression for $e_{TA}$ can be used for those cases where a particular hardware configuration is being considered. The dependence of $e_{TA}$ on the $(S/N)_r$, can be made more explicit if we write it as:

$$e_{TA} = \zeta \left( \frac{S}{N} \right)$$

$$\zeta = \frac{(NF_{Rx})(N_0)(BW)}{(G_{ant})(\eta_{amp})(R_{bit})} \left( \frac{L_0}{d_0} \right)^\alpha$$
It is important to recognize this dependence explicitly since it highlights the relationship between $e_{TA}$ and the probability of bit error $p$, which depends on $(S/N)_r$. In this paper, we have assumed a variable transmission power scenario which means that the radio can dynamically adjust its transmission power so a desired $(S/N)_r$ at the receiver is guaranteed.

### III. MULTIHOP ENERGY CONSUMPTION

Let us consider a linear sensor array as shown in Fig. 2 which has also been used in other studies [3]:

![Linear topology](image)

Figure 2. Linear topology.

For link $i$ assume that the probability of bit error is $p_i$. Assume a packet length of $m$ bits. For the analysis below we assume that a Forward Error Correction (FEC) mechanism is being used. Let us then call $P_{link}(i)$ the probability of receiving a packet with uncorrectable errors. The conventional use of FEC is that at each hop the packet is accepted and delivered to the next stage, which in this case is forwarding it to the next node downstream. Assuming a variable transmission power mode the energy consumed in sending a packet from the $n_{th}$ node to the sink using a multihop routing that uses the downstream neighbor as a relay node is:

$$E_{linear} = m\left[ e_{TC} + e_{RA}(d_i)^\alpha + \sum_{i=2}^{n} e_{TC} + e_{RC} + e_{RA}(d_i)^\alpha \right]$$

(7)

and the probability of the packet arriving at the $n$th sink node with no errors is:

$$P_e = \prod_{i=1}^{n} (1 - P_{link}(i))$$

(8)

It can be easily shown that $E_{linear}$ is minimized when all the distances are the same, i.e. $d_i = D/n$, and thus

$$E_{linear} = m\left[ n(e_{TC} + e_{RC}) - e_{RC} + e_{RA}D^\alpha \right]$$

(9)

where $n$ is the number of hops. The optimal number of hops can be calculated as [2]:

$$n_{opt} = \left\lfloor \frac{D}{d_{char}} \right\rfloor$$

(10)

where the characteristic distance $d_{char}$ is defined as:

$$d_{char} = \sqrt[\alpha]{\frac{e_{TC} + e_{RC}}{e_{RA}(\alpha - 1)}}$$

(11)

where $\alpha$ is the path loss exponent of the channel which is typically between 2 and 4. In this paper we use $\alpha=3$.

### IV. WIRELESS CHANNEL MODEL

The wireless channel characteristics affect the reliability of the communication link. Bit error rate (BER) is a very useful metric when comparing the energy consumption of various communication methods. A common modulation technique used in WSNs is a non-coherent (envelope or square-law) detector with binary orthogonal FSK signals. For this case the probability of bit error for a non-fading channel is [4]:

$$p(\gamma_b) = \frac{1}{2} e^{-\gamma_b/2}$$

(12)

For typical sensor networks it is more reasonable to assume a Rayleigh slow fading channel attenuation. In this case,

$$p_{FSK} = \frac{1}{2 + \gamma_b}$$

(13)

where $\gamma_b$ is the average signal-to-noise ratio.

### V. LINEAR CODES

Common codes are used in this study so that the trade-off between coding overhead and energy consumption per information bit can be illustrated. We use linear block codes of the form $(m, k, d_{min})$, where $m$ is the length of code word, $k$ is the number of information bits and $d_{min}$ is the minimum distance of the code. Parameters for the studied codes are shown in Table 1.

<table>
<thead>
<tr>
<th>Code</th>
<th>$m$</th>
<th>$k$</th>
<th>$d_{min}$</th>
<th>Code rate</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>0.57</td>
<td>1</td>
</tr>
<tr>
<td>Golay</td>
<td>23</td>
<td>12</td>
<td>7</td>
<td>0.52</td>
<td>3</td>
</tr>
<tr>
<td>Shortened Hamming</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Extended Golay</td>
<td>24</td>
<td>12</td>
<td>8</td>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Probabilities of code word errors for the different codes depend on the channels’ bit error rate and the properties of the codes. Since a variable transmission power mode is being assumed the probability of the bit error for each link is fixed and thus $P_e = (1 - P_{link})^n$. The value of $P_{link}$ will depend on the received signal to noise ratio as well as on the modulation and coding method used. Consider a linear code $(m, k, d)$ is being used. For FSK-modulation with non-coherent detection and assuming ideal interleaving the probability of a code word being in error is bounded by [4]:

$$P_M < \sum_{i=2}^{M} \left( \frac{2w_i - 1}{2 + \gamma_b} \right) \gamma_b^{d_{min}}$$

(14)

where $w_i$ is the weight of the $i$th code word and $M=2^k$. A simpler bound is:

$$P_M < (M - 1)[4 p_{FSK} (1 - p_{FSK})]^{d_{min}}$$

(15)

In the case being discussed here $P_{link} = P_M$ and the probability of packet error for the multihop scenario of Equation 10 can be written as:
\[ P_e = 1 - P_c = 1 - (1 - p_{link})^n = 1 - (1 - P_M)^n \]  
\[ < 1 - (1 - (2^k - 1)4P_{FSK}(1 - P_{FSK})^{y_{\text{min}}})^n \]  
Equation (16)

The probability of successful transmission of a single code word is

\[ P_{\text{success}} = (1 - P_c) \]  
Equation (17)

The expected energy consumption per information bit is defined as:

\[ E_{\text{i-bit}} = \frac{E_{\text{linear}}}{(k)(P_{\text{success}})} \]  
Equation (18)

VI. RESULTS

The following numerical analysis was made by observing the effects of coding on the energy consumption of multihop WSNs. For these calculations, the size \( D \) of the linear array was assumed to be 1000 meters. The energy consumption per information bit for different codes using different numbers of nodes was calculated using the equations from the previous sections.

Fig. 3 shows \( d_{\text{char}} \) as a function of bit error probability \( P_{FSK} \).

Fig. 4 shows \( E_{\text{i-bit}} \) for the different codes when the number of nodes is 10. In this case the distance of one hop is 100 meters which is about three times longer than the optimal distance between two neighbor nodes.

Fig. 5 shows the energy consumption when \( n = 30 \). This makes the distance of one hop to be near the optimal value. In this case the energy efficiency of the various codes has increased. The energy efficiency of the simpler codes start to decrease when \( P_{FSK} \) is above 0.01.

In this case the large number of hops increases the probability of code word error. This effect is more clearly seen in Fig. 6 where the number of hops is 60. For \( P_{FSK} \) equal to 0.01, the Golay codes are better because they have a stronger error correction capability. Fig. 6 also shows that energy efficiency of all the codes decreases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( NF_{Rx} )</td>
<td>10dB</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>-173.8 dBm/Hz or ( 4.17 * 10^{-21} ) J</td>
</tr>
<tr>
<td>( R_{\text{bit}} )</td>
<td>115.2 Kbits</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.3 m</td>
</tr>
<tr>
<td>( G_{\text{ant}} )</td>
<td>-10dB or 0.1</td>
</tr>
<tr>
<td>( \eta_{\text{amp}} )</td>
<td>0.2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>3</td>
</tr>
<tr>
<td>( BW )</td>
<td>Depends on the modulation method. For FSK-modulation, it can be assumed to be the same as ( R_{\text{bit}} )</td>
</tr>
<tr>
<td>( \epsilon_{\text{TC}} )</td>
<td>50nJ/bit</td>
</tr>
<tr>
<td>( \epsilon_{\text{RC}} )</td>
<td>50nJ/bit</td>
</tr>
</tbody>
</table>

Figure 3. Characteristic distance as a function of bit error probability \( P_{FSK} \).

Figure 4. Expected energy consumption per information bit, \( n=10 \).

Figure 5. Expected energy consumption per information bit, \( n=30 \).

Figure 6. Expected energy consumption per information bit, \( n=60 \).
when compared to the 30-hop case because the length of a single hop is below the optimal distance.

Fig. 7 shows the energy efficiency of the (7,4,3) code for different scenarios of $n$. It is observed that 30 hops is the most energy efficient distance because in this case the length of one hop is about 33 meters which is near the optimal distance of communication between two neighbor nodes as it is shown in Fig. 3.

When the separation between nodes is close to $d_{\text{crit}}$ the energy required by the transmitter amplifier to achieve the desired signal to noise ratio at the receiver is optimized. When the number of hops is increased to 60 the length of a single hop is about 16.7 meters. As the distance of a single hop decreases, the transmission power needed for a desired SNR in the receiver also decreases but as the number of hops increases the code word error probability increases, which in turn decreases the overall energy efficiency. When the one hop distance increases, as in the case of 10 hops, the transmission power also increases, but then only 10 transmissions are needed. Fig. 7 shows that for the case of the (7,4,3) code it is better to have many hops than few when the value of $p_{\text{FSK}}$ is less than 0.02. When this value is over 0.025, few hops are better than many hops. The reason for this is the increased code word error probability as the number of hops increases.

Fig. 8 illustrates that with the extended Golay code, it is better to use many hops than too few. The (24,12,8) code has error correction capability $t=3$ and so its code word error is not severely affected when the number of hops is high.

VII. CONCLUSIONS

In this paper we have analyzed how different channel codes can affect the energy consumption in multihop wireless sensor networks. Energy efficiencies of the codes were calculated with an analytical radio model when the number of hops was near optimal as well as below and above the optimal value. The results show that depending on the channel conditions, the efficiencies of the codes vary. In good channel conditions, simpler codes work better. Error control is more useful when the channel conditions get worse. The number of hops also has an effect on the energy consumption. When the number of hops increases, the importance of error correction control also increases. The results show that it is better to keep the number of hops above rather than below the optimal number of hops when reliable coding is used.

Our model assumed that the transmitter can dynamically adjust its transmission power so that the desired signal to noise ratio is guaranteed at the receiver’s demodulator. This is not a realistic assumption because in current transceivers, a fixed transmission power scenario is used. The codes used in this work are quite simple with short block lengths. Further studies need to be done to address the fixed transmission power scenario and also to find more efficient coding techniques for practical WSN applications.

REFERENCES

