Beamforming and Adaptive Antennae

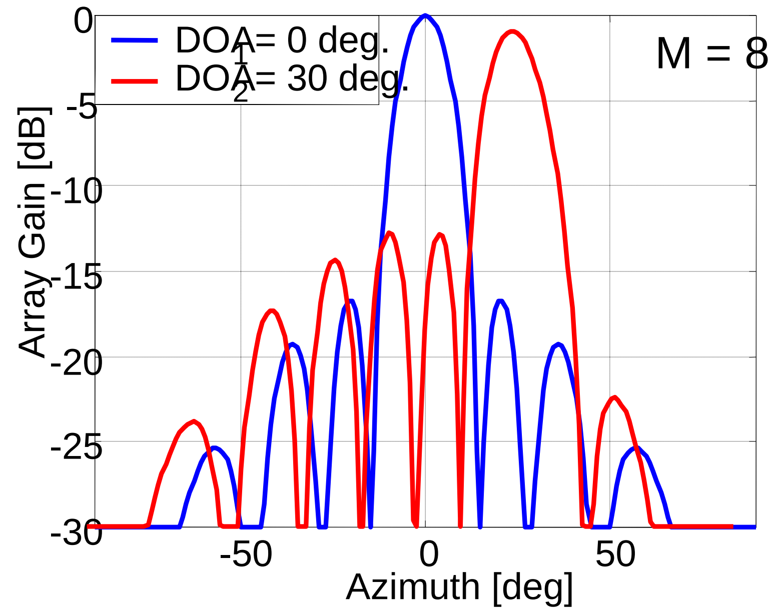
Markku Juntti and Juha Ylitalo

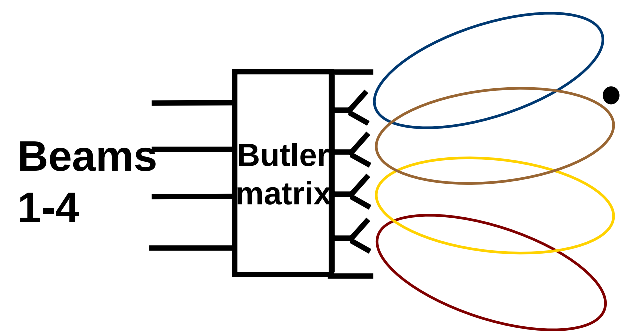
Contents

Beamforming: phased array

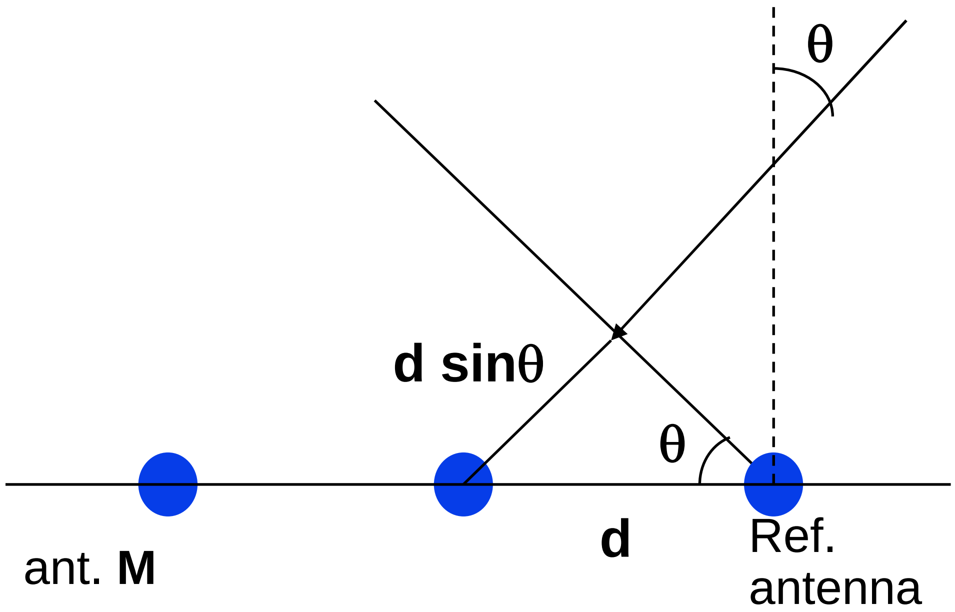
- Beam steering = phasing the antenna array elements
- Assumptions:
  - – small angular spread (macro cell)
  - □ $\lambda/2$ ant. element separation
    - --> correlated antennas
    - --> Rxx ~diagonal
- "Fixed spatial filter"
  - – based on estimated DOA of desired user
  - – DOA tracking (slow vs. channel tracking)
  - – fixed beams by e.g. Butler matrix feasible
  - – interference suppressed by the beam pattern

DOA = Direction of Arrival





Beamforming: array response vector



Uniform Linear Array :

antenna M separated by **(m-1)d** from ref. antenna

d= $\lambda/2$

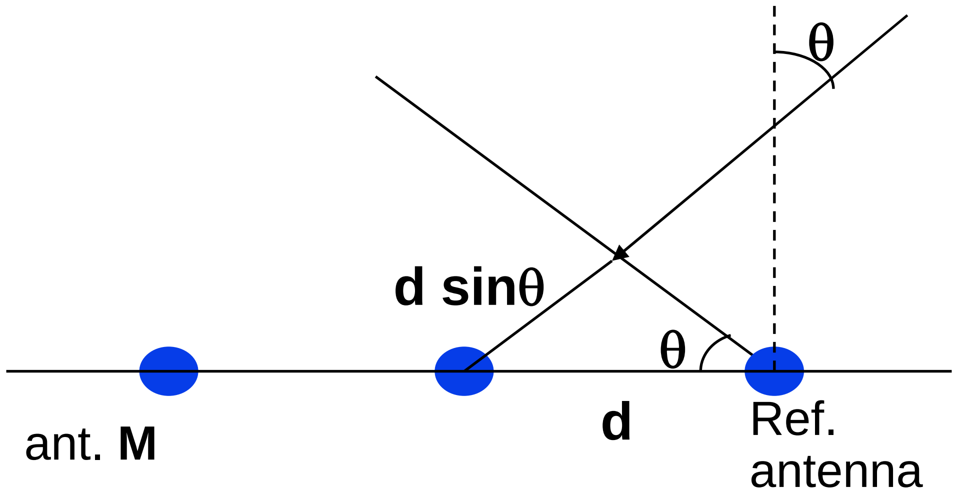
- Array response vector:

$$a(\theta) = [1, \; e^{-j2\pi d \sin(\theta)/\lambda}, \; e^{-j4\pi d \sin(\theta)/\lambda} \; \ldots \; e^{-j2\pi(M-1)d \sin(\theta)/\lambda}]^T$$

$$= [1 \; e^{-j\pi \sin(\theta)} \; e^{-j2\pi \sin(\theta)} \; \ldots \; e^{-j\pi(M-1) \sin(\theta)}]^T$$

when **d=λ/2** ; T denotes transpose

Beamforming: azimuth power spectrum



- Power spectrum as a function of azimuth angle θ

$$P(\theta) = [a(\theta)^H x(\theta)]^2 = a(\theta)^H \{x(\theta)x(\theta)^H\} a(\theta)$$

$$\Rightarrow \quad P(\theta) = a(\theta)^H R_{xx} a(\theta)$$

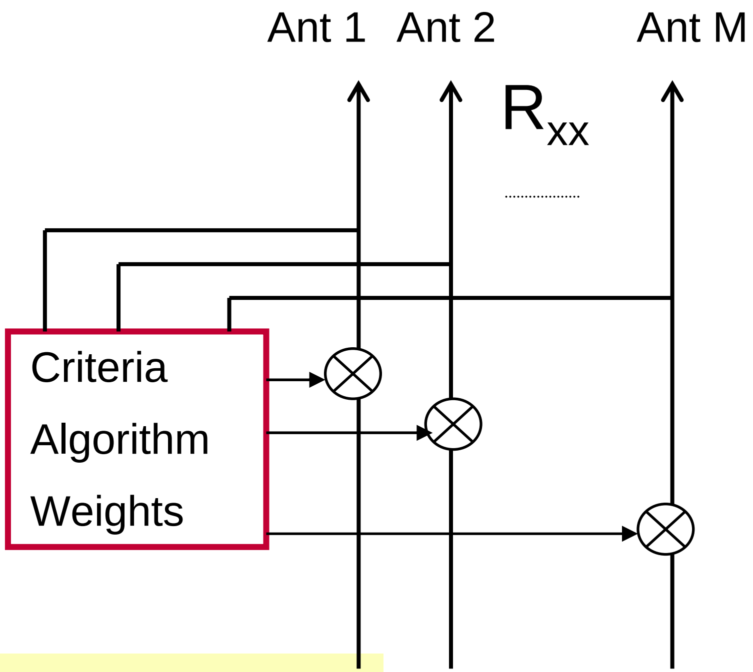
H denotes here the complex conjugate transpose

DOA resolution is limited by the array aperture: $\Delta\theta \sim 96\text{deg}/M$ with ULA

Adaptive antennas

" Optimal multi-channel filtering"

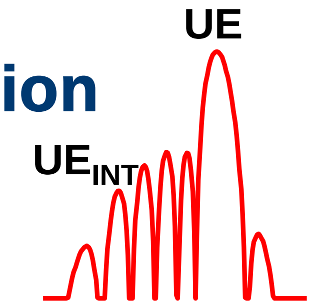
- Optimisation criteria
  - MMSE, ML, MV ...
- Optimal combining weights
  - typically in the form: $\alpha R_{xx}^{-1} h^*$ ...
- Adaptive antenna algorithms
  - LMS, RLS, DMI, ...



Here we consider only optimal ***spatial*** combining algorithms:

- Narrow-band assumption
- Wide-band assumed only in 2D Rake receiver sense
- Full-scale space-time equalisation will not be treated here

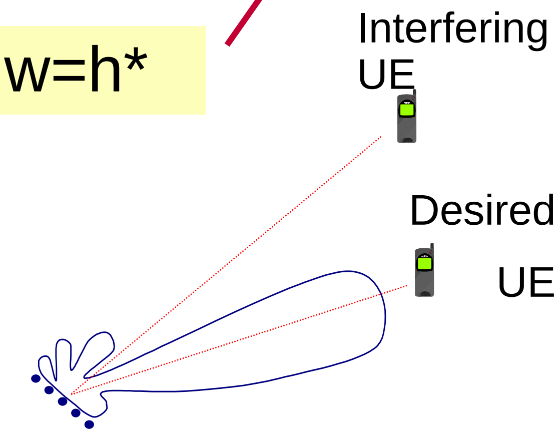
Adaptive antenna algorithms: Introduction



- Two basic cases:
  - Beamforming with high correlated antennas
  - Diversity reception with low correlated antennas

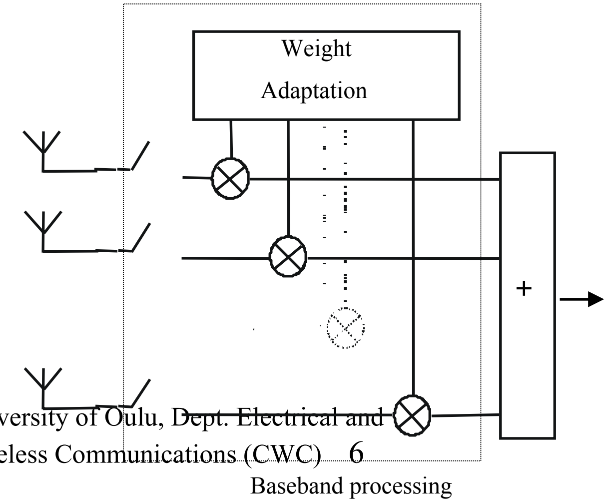
- Beamforming can utilise directional information
  - allows simple DoA based algorithms: only DOA needed to be estimated
  - optimal in Line-Of-Sight, AWGN case
  - DoA based algorithms apply also to transmission
  - allows also optimum combining algorithms

$$w = h^*$$



- Diversity reception
  - optimum combining
  - each antenna weight has to be separately estimated
  - antenna weight proportional to SINR

$$w = R_{xx}^{-1} h^*$$



Optimisation criteria

- Assumptions:

  - Signal and interference processes are uncorrelated --> different propagation paths
  - Signal and interference processes are stationary --> during estimation of weights
  - Signal and interference processes are additive --> linear radio channel

Performance of an adaptive algorithm depends on the statistical character of the desired and interfering signal in the temporal and spatial domain:

- fading (temporal correlation)

- correlation between antenna elements

"Optimum combining algorithms can suppress correlated interference at antenna array"

Idealised or perturbed propagation conditions?

Optimisation criteria, cont'd

Optimisation of antenna combining weights based on

- Maximum Signal-to-Interference-and-Noise criterion (SINR)
- Least Mean Square criterion (LMS) --MMSE
- Maximum Likelihood criterion (ML)
- Minimum Variance criterion (MV)

It is interesting to note that all these criteria lead to similar solution of optimal antenna array weight vector which can be described as:

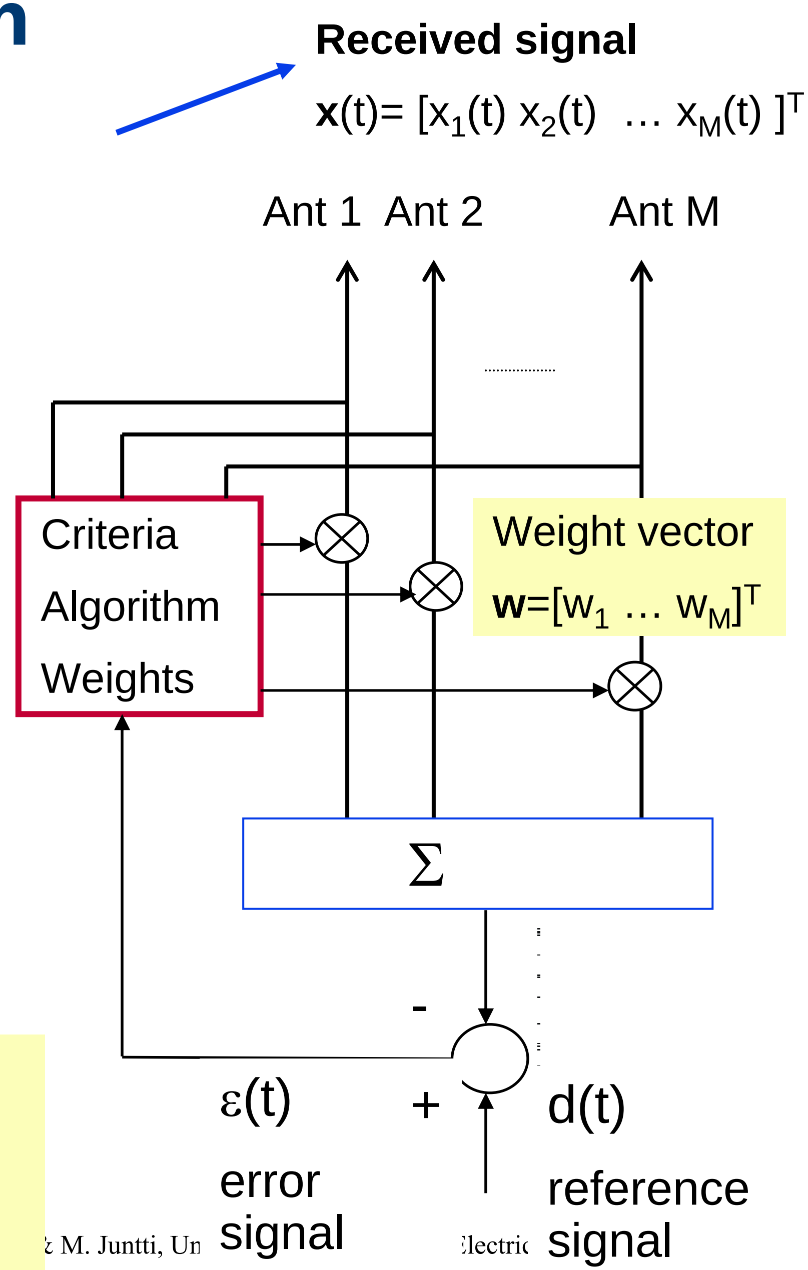
$$w_{opt} = \alpha R_{xx}^{-1} h^*$$

in which α is a scalar scaling factor, $\mathbf{w}_{opt}$ the optimal antenna weight vector

$\mathbf{R}_{xx}^{-1}$ is the spatial correlation matrix and **h** represents the array response vector of the desired signal

MMSE criterion

- Widrow 1960



Error signal:

$$\varepsilon(t) = d(t) - \mathbf{w}^* \mathbf{x}(t)$$

Squared error:

$$\varepsilon(t)^2 = d(t)^2 - 2d(t)\mathbf{w}^*\mathbf{x}(t) + \mathbf{w}^*\mathbf{x}(t)\mathbf{x}^*(t)\mathbf{w}$$

Expected error value:

$$E\{\varepsilon(t)^2\} = \underline{d(t)}^2 - 2\mathbf{w}^*\mathbf{r}_{xd}(t) + \mathbf{w}^*\mathbf{R}_{xx}\mathbf{w}$$

$\mathbf{R}_{xx} = E\{\mathbf{x}\,\mathbf{x}^*\}$, is a MxM spatial correlation matrix

$\mathbf{r}_{xd} = E\{d(t)\,\mathbf{x}(t)\}$, is a Mx1 column vector (= h , ch.est.)

$\underline{d(t)}^2$ = signal power

MMSE ...

*- selecting optimal **w** to minimise error: set gradient of* E{ε(t)²} *with respect to **w** to zero*

$$\nabla_w \{<(\varepsilon^2)>\} = -2\mathbf{r}_{xd}(t) + 2\mathbf{R}_{xx}\mathbf{w} = 0$$

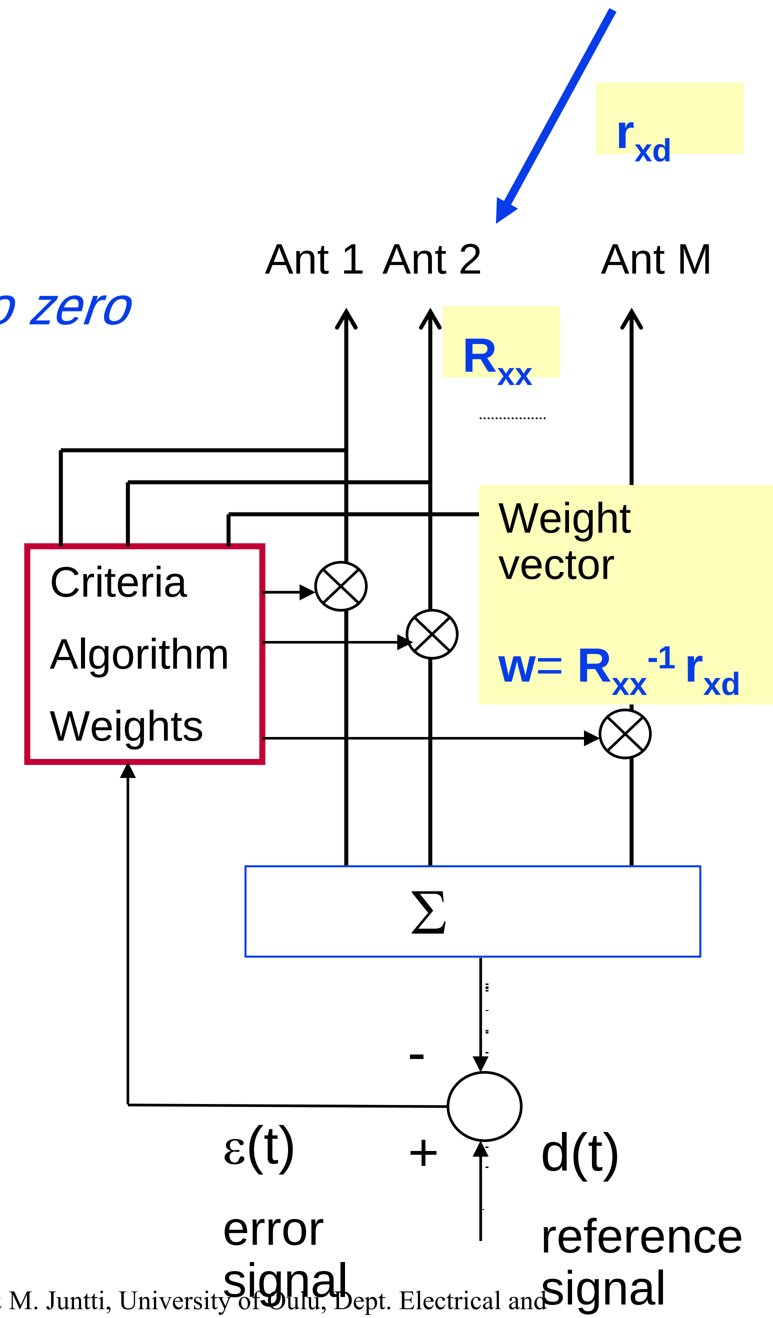
Thus optimal weight vector is obtained from:

$$\mathbf{R}_{xx}\mathbf{w}_{opt} = \mathbf{r}_{xd}$$

and

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{xd}$$

Above equation is known as Wiener-Hopf equation in matrix form. It is usually referred as optimum Wiener solution



Optimal combining weights

- **MMSE (Wiener solution):**

$$w_{MMSE} = R_{xx}^{-1} v^* = \frac{S}{1 + S v^* R_{nn}^{-1} v} R_{nn}^{-1} v^*$$

- **Max SINR:**

$$w_{SINR} = \alpha R_{nn}^{-1} v^*$$

- **Maximum Likelihood:**

$$w_{ML} = \frac{1}{v^* R_{nn}^{-1} v} R_{nn}^{-1} v^*$$

- **Minimum variance:**
  - Direction of desired signal known

$$\boldsymbol{w}_{MV} = \frac{\boldsymbol{R}_{nn}^{-1}\boldsymbol{1}}{\boldsymbol{1}^* \boldsymbol{R}_{nn}^{-1}\boldsymbol{1}}$$

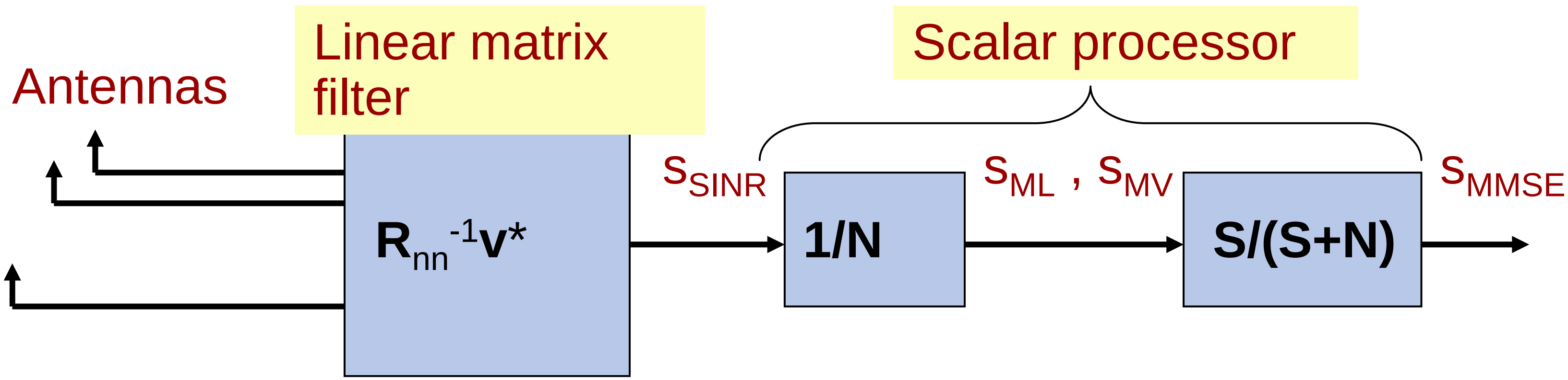
S= signal power, v* = array response vector of the desired signal, R_{nn}= interference+noise covariance matrix, R_{xx}= total received signal (signal+interference+noise) covariance matrix, v=**1** for known signal weights in MV

Note:

here v = r$_{xd}$

x(t) = s(t)**v + n(t)**

Optimal combining weights



- **Notations used above:**

$\mathbf{x}(t) = s(t)\mathbf{v} + \mathbf{n}(t)$

$\mathbf{R}_{xx} = E\{\mathbf{x}(t)\,\mathbf{x}^*(t)\}$

$\mathbf{R}_{nn} = E\{\mathbf{n}(t)\,\mathbf{n}^*(t)\}$

$\mathbf{R}_{ss} = E\{\mathbf{s}(t)\,\mathbf{s}^*(t)\}$

$\mathbf{s}(t) = s(t)\mathbf{v}$

$\mathbf{R}_{xx} = \mathbf{R}_{ss} + \mathbf{R}_{nn}$

Note:

$\mathbf{n}(t)$ includes both interference and AWGN: $\mathbf{n}(t)=\mathbf{I}(t)+\mathbf{N}_0$

$\mathbf{v}$ is the array response vector corresponding to desired signal direction

Adaptive antenna algorithms

- Adaptive antenna algorithms solve the optimal antenna weights for (rapidly) changing radio channel state
- Different criteria (MMSE, max SINR, ML, MV) lead to the same solution

$$w_{opt} = \alpha R_{nn}^{-1} r_{xd}^{*}$$

- Task is to estimate $\mathbf{R}_{nn}$ and $\mathbf{r}_{xd}$ and find a simple and robust (direct or iterative) way to invert $\mathbf{R}_{nn}$
- $\mathbf{R}_{nn}$ characteristics have an important role in the adaptation

For more details about matrix inversion, see

S. Haykin: "Adaptive filter theory", Prentice Hall

- Typical adaptive antenna algorithms:

- LMS

- DMI

- RLS

Example: DMI algorithm

- Direct matrix inversion

$$w_{opt} = R_{xx}^{-1} r_{xd}^*$$

 only estimates of $\mathbf{R}_{nn}$ and $\mathbf{r}_{xd}$ are known:

$$E\{R_{xx}\} = \hat{R}_{xx} = \frac{1}{K}\sum_{k=1}^{K} x(k)x^*(k)$$

$$E\{r_{xd}\} = \hat{r}_{xd} = \frac{1}{K}\sum_{k=1}^{K} x(k)d^*(k)$$

- Thus we get

$$\hat{w}_{opt} = \hat{R}_{nn}^{-1}\hat{r}_{xd}$$

if R_{nn} can be estimated (α =1)

or if only R_{xx} can be estimated

$$\hat{w}_{opt} = \hat{R}_{xx}^{-1}\hat{r}_{xd}$$

DMI algorithm

- Example: GSM burst

58 bit	26 bit (**d**)	58 bit
Data	Training sequence	Data

$$E\{R_{xx}\} = \hat{R}_{xx} = \frac{1}{K}\sum_{k=1}^{K} x(k)x^*(k)$$

$$E\{r_{xd}\} = \hat{r}_{xd} = \frac{1}{K}\sum_{k=1}^{K} x(k)d^*(k)$$

$$\hat{w}_{opt} = \hat{R}_{xx}^{-1}\hat{r}_{xd}$$

- Procedure
  - calculate E{$\mathbf{R}_{xx}$} over the whole burst (K=148)
  - calculate E{$\mathbf{r}_{xd}$} over the known symbols (K=26)

  (actually E{$\mathbf{r}_{xd}$} = E{**h**}= estimate of the impulse response)

- E{$\mathbf{R}_{xx}$} and E{$\mathbf{r}_{xd}$} are calculated for each burst
- E{$\mathbf{R}_{xx}$} and E{$\mathbf{r}_{xd}$} can be averaged over several bursts
- If $\mathbf{r}_{xd}$ is known and $\mathbf{R}_{nn}$ can be estimated:
  => K > 3M (roughly)

$$E\left\{\frac{snr}{snr_{opt}}\right\} \cong \frac{K+2-M}{K+1}$$

K=no of independent samples, M= no of antennas

Summary

- Simple beamforming (beampointing) is simple and robust and applies well to FDD systems
- Adaptive antenna algorithms can be applied to interference suppression
- Adaptive antenna algorithms/ optimum combining is required for good MIMO performance: ”space-time equalisation”

References

- Monzingo and Miller, Introduction to Adaptive Arrays", Wiley, 1980.
- Winters, J. "Optimum Combining in Digital Mobile Radio with Cochannel Interference", Selected Areas in Communications, IEEE Journal on ,Volume: 2 , Issue: 4 , Jul 1984, Pages:528 - 539
- J. C. Liberti. and T. S. Rappaport, Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications, Prentice Hall, Upper Saddle River, NJ, 1999.
- Ylitalo, J.; Tiirola, E.; " Performance evaluation of different antenna array approaches for 3G CDMA uplink", Vehicular Technology Conference Proceedings, 2000. VTC 2000-Spring Tokyo. 2000 IEEE 51st ,Volume: 2 , 15-18 May 2000, Pages: 883 - 887 vol.2