

Beamforming and Adaptive Antennae

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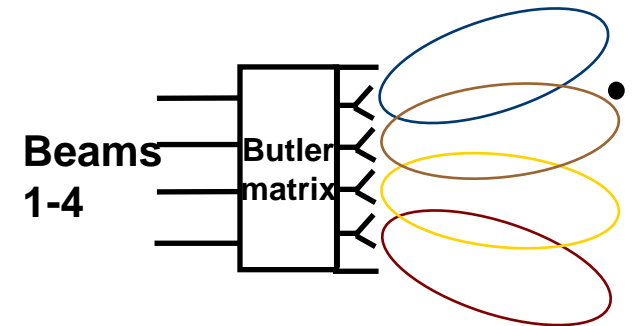
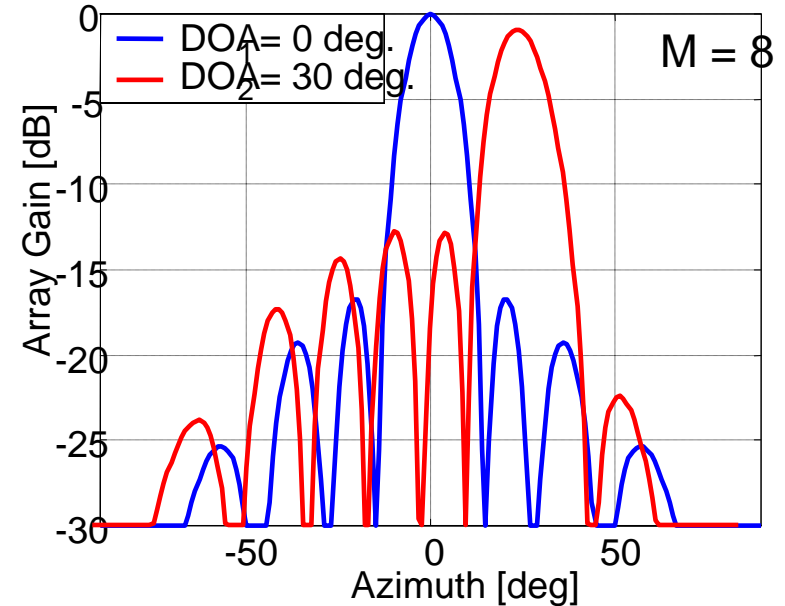
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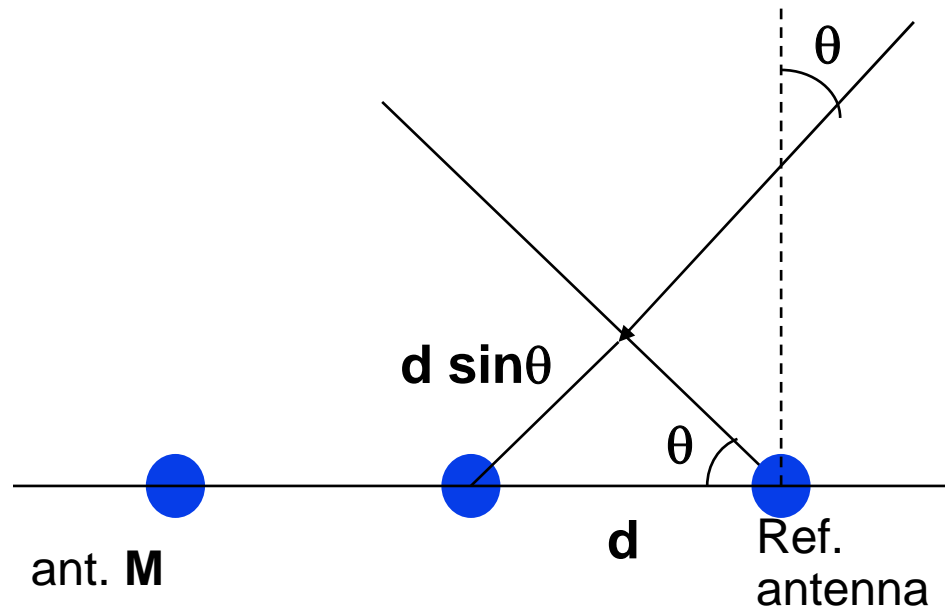
Beamforming: phased array

- Beam steering = phasing the antenna array elements
- Assumptions:
 - small angular spread (macro cell)
 - $\lambda/2$ ant. element separation
 - > correlated antennas
 - > R_{xx} ~diagonal
- "Fixed spatial filter"
 - based on estimated DOA of desired user
 - DOA tracking (slow vs. channel tracking)
 - fixed beams by e.g. Butler matrix feasible
 - interference suppressed by the beam pattern

DOA = Direction of Arrival



Beamforming: array response vector



Uniform Linear Array :

antenna M separated by $(m-1)d$ from ref. antenna

$$d = \lambda/2$$

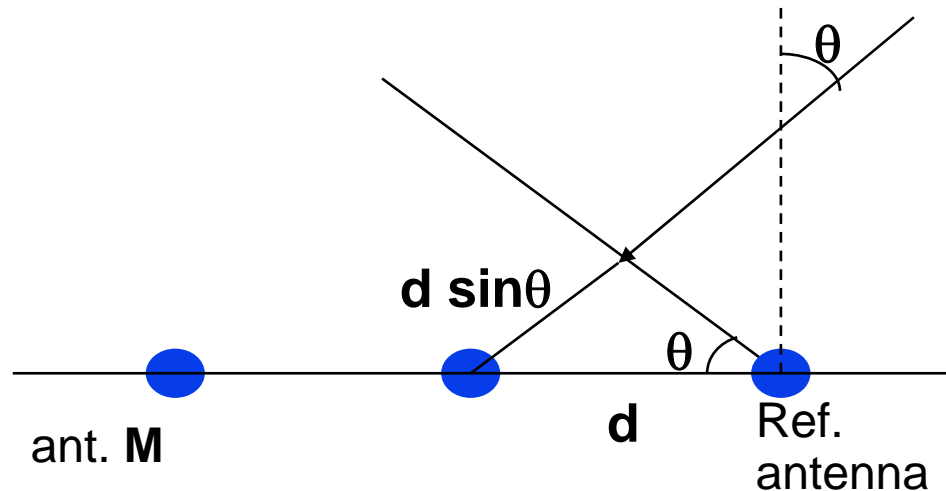
- Array response vector:

$$a(\theta) = [1, e^{-j2\pi d \sin(\theta)/\lambda}, e^{-j4\pi d \sin(\theta)/\lambda} \dots e^{-j2\pi(M-1)d \sin(\theta)/\lambda}]^T$$

$$= [1 \ e^{-j\pi \sin(\theta)} \ e^{-j2\pi \sin(\theta)} \ \dots \ e^{-j\pi(M-1) \sin(\theta)}]^T$$

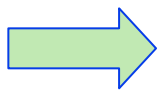
when $d = \lambda/2$; T denotes transpose

Beamforming: azimuth power spectrum



- Power spectrum as a function of azimuth angle θ

$$P(\theta) = [a(\theta)^H x(\theta)]^2 = a(\theta)^H \{x(\theta)x(\theta)^H\} a(\theta)$$



$$P(\theta) = a(\theta)^H R_{xx} a(\theta)$$

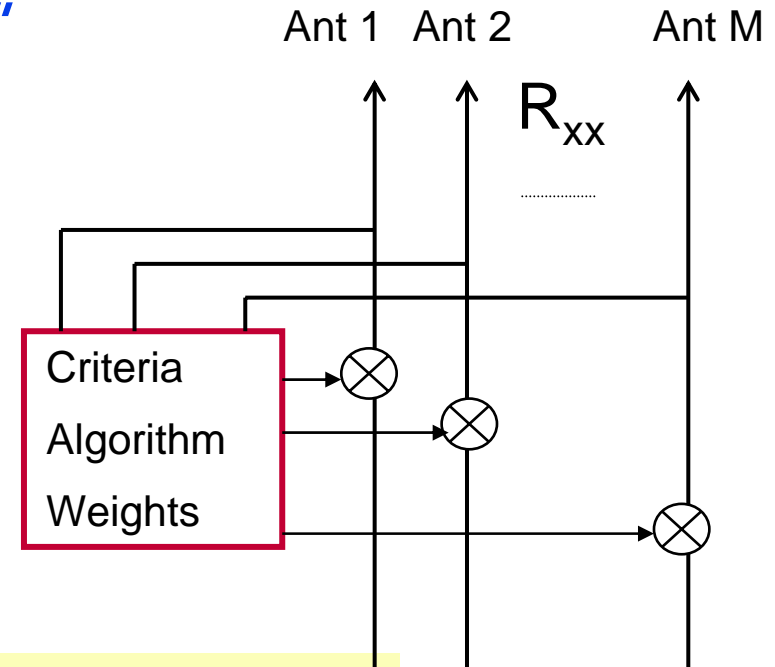
H denotes here the complex conjugate transpose

DOA resolution is limited by the array aperture: $\Delta\theta \sim 96\text{deg}/M$ with ULA

Adaptive antennas

"Optimal multi-channel filtering"

- Optimisation criteria
 - MMSE, ML, MV ...
- Optimal combining weights
 - typically in the form: $\alpha R_{xx}^{-1} h^* \dots$
- Adaptive antenna algorithms
 - LMS, RLS, DMI, ...



Here we consider only optimal *spatial* combining algorithms:

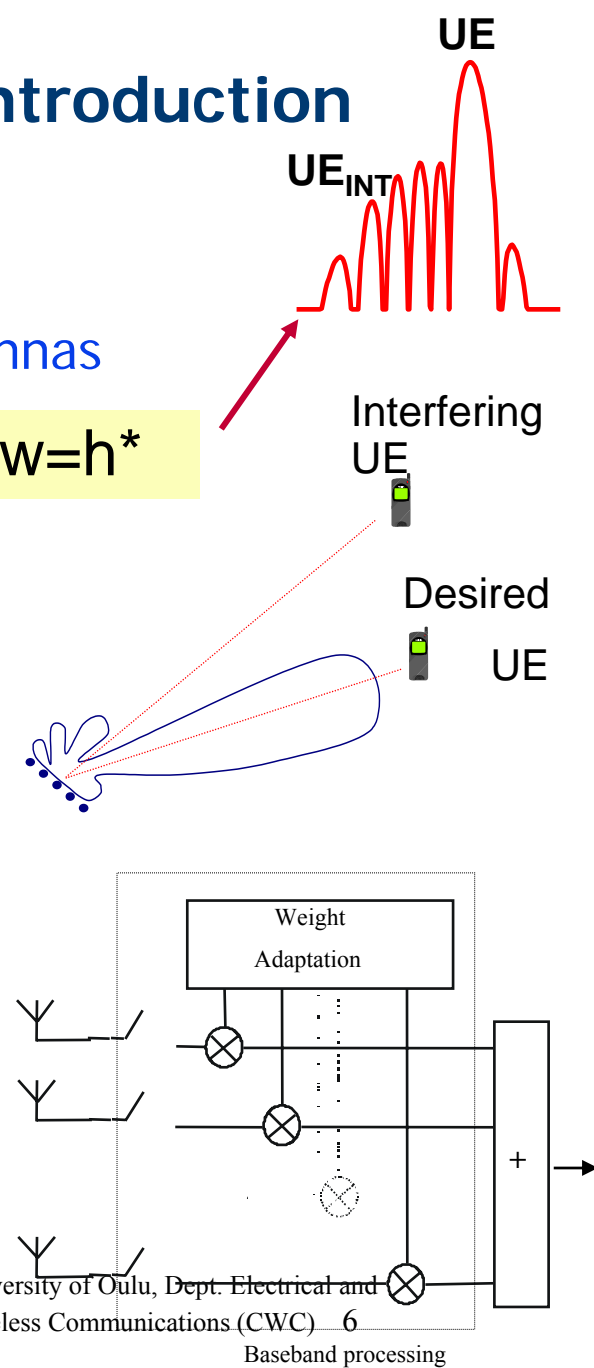
- Narrow-band assumption
- Wide-band assumed only in 2D Rake receiver sense
- Full-scale space-time equalisation will not be treated here

Adaptive antenna algorithms: Introduction

- Two basic cases:
 - Beamforming with high correlated antennas
 - Diversity reception with low correlated antennas
- Beamforming can utilise directional information
 - allows simple DoA based algorithms: only DOA needed to be estimated
 - optimal in Line-Of-Sight, AWGN case
 - DoA based algorithms apply also to transmission
 - allows also optimum combining algorithms
- Diversity reception
 - optimum combining
 - each antenna weight has to be separately estimated
 - antenna weight proportional to SINR

$$w = h^*$$

$$w = R_{xx}^{-1} h^*$$



Optimisation criteria

- Assumptions:

- Signal and interference processes are uncorrelated --> different propagation paths
- Signal and interference processes are stationary --> during estimation of weights
- Signal and interference processes are additive --> linear radio channel

Performance of an adaptive algorithm depends on the statistical character of the desired and interfering signal in the temporal and spatial domain:

- fading (temporal correlation)
- correlation between antenna elements

"Optimum combining algorithms can suppress correlated interference at antenna array"



Idealised or perturbed propagation conditions?

Optimisation criteria, cont'd

Optimisation of antenna combining weights based on

- Maximum Signal-to-Interference-and-Noise criterion (SINR)
- Least Mean Square criterion (LMS) --MMSE
- Maximum Likelihood criterion (ML)
- Minimum Variance criterion (MV)

It is interesting to note that all these criteria lead to similar solution of optimal antenna array weight vector which can be described as:

$$\mathbf{w}_{\text{opt}} = \alpha \mathbf{R}_{\text{xx}}^{-1} \mathbf{h}^*$$

in which α is a scalar scaling factor, \mathbf{w}_{opt} the optimal antenna weight vector $\mathbf{R}_{\text{xx}}^{-1}$ is the spatial correlation matrix and \mathbf{h} represents the array response vector of the desired signal

MMSE criterion

- *Widrow 1960*

Error signal:

$$\varepsilon(t) = d(t) - \mathbf{w}^* \mathbf{x}(t)$$

Squared error:

$$\varepsilon(t)^2 = d(t)^2 - 2d(t)\mathbf{w}^* \mathbf{x}(t) + \mathbf{w}^* \mathbf{x}(t)\mathbf{x}^*(t)\mathbf{w}$$

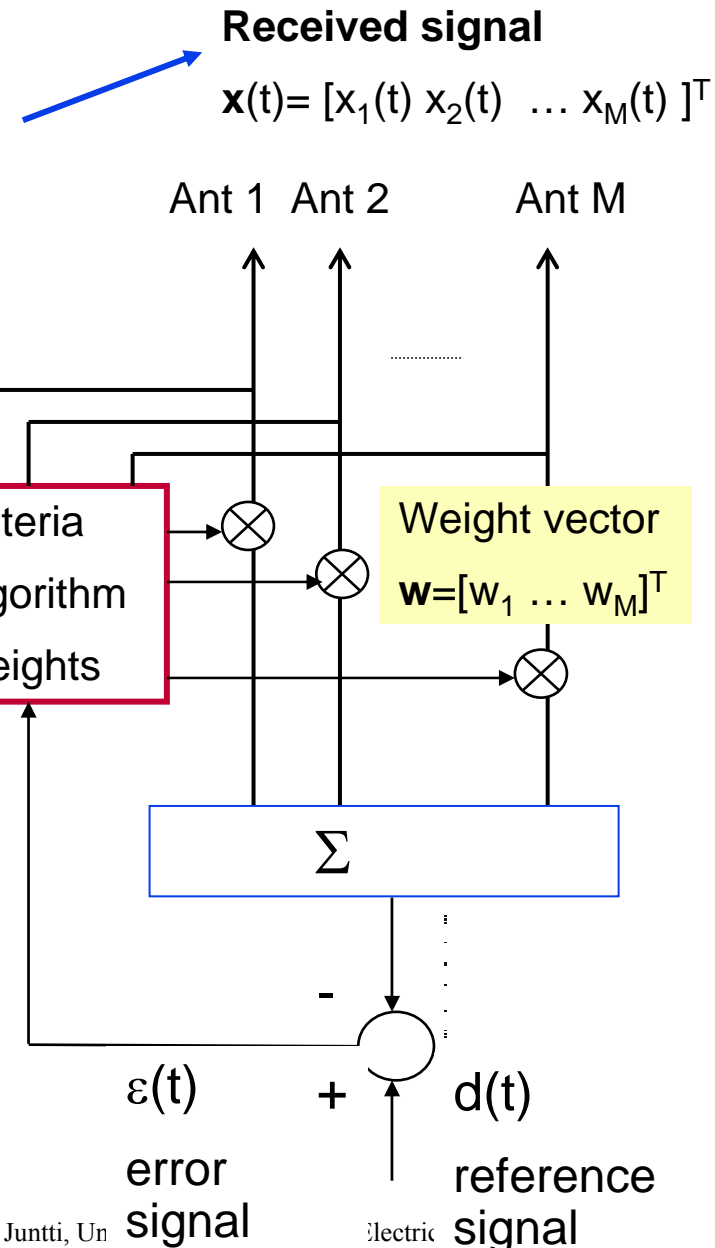
Expected error value:

$$E\{\varepsilon(t)^2\} = \underline{d(t)}^2 - 2\mathbf{w}^* \mathbf{r}_{xd}(t) + \mathbf{w}^* \mathbf{R}_{xx} \mathbf{w}$$

$\mathbf{R}_{xx} = E\{\mathbf{x} \mathbf{x}^*\}$, is a $M \times M$ spatial correlation matrix

$\mathbf{r}_{xd} = E\{d(t) \mathbf{x}(t)\}$, is a $M \times 1$ column vector (= \mathbf{h} , ch.est.)

$\underline{d(t)}^2 =$ signal power



MMSE ...

- selecting optimal \mathbf{w} to minimise error: set gradient of $E\{\varepsilon(t)^2\}$ with respect to \mathbf{w} to zero

$$\nabla_{\mathbf{w}} \{ \langle (\varepsilon^2) \rangle \} = -2\mathbf{r}_{\text{xd}}(t) + 2\mathbf{R}_{\text{xx}}\mathbf{w} = \mathbf{0}$$

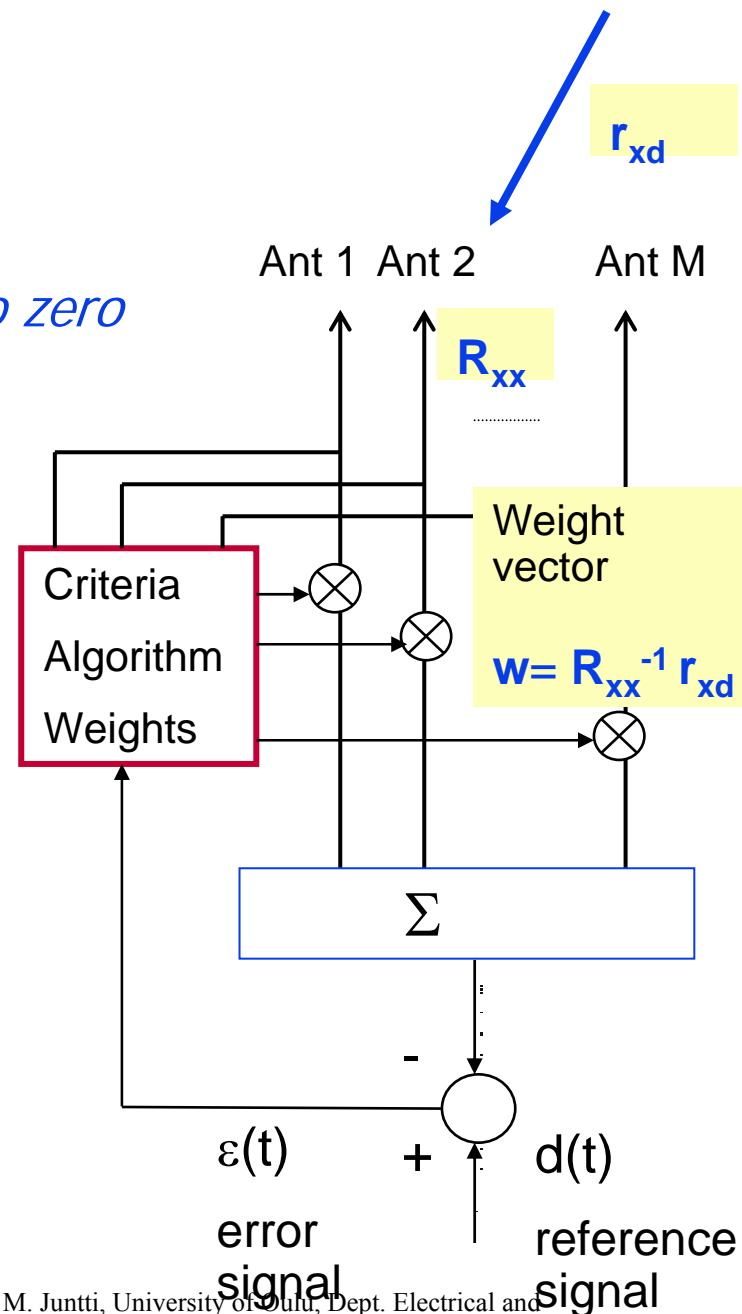
Thus optimal weight vector is obtained from:

$$\mathbf{R}_{\text{xx}}\mathbf{w}_{\text{opt}} = \mathbf{r}_{\text{xd}}$$

and

$$\mathbf{w}_{\text{opt}} = \mathbf{R}_{\text{xx}}^{-1} \mathbf{r}_{\text{xd}}$$

Above equation is known as Wiener-Hopf equation in matrix form. It is usually referred as optimum Wiener solution



Optimal combining weights

- **MMSE (Wiener solution):**

$$w_{MMSE} = R_{xx}^{-1} v^*$$

$$= \frac{S}{1 + S v^* R_{nn}^{-1} v} R_{nn}^{-1} v^*$$

- **Max SINR:**

$$w_{SINR} = \alpha R_{nn}^{-1} v^*$$

- **Maximum Likelihood:**

$$w_{ML} = \frac{1}{v^* R_{nn}^{-1} v} R_{nn}^{-1} v^*$$

- **Minimum variance:**
- Direction of desired signal known

$$w_{MV} = \frac{R_{nn}^{-1} \mathbf{1}}{\mathbf{1}^* R_{nn}^{-1} \mathbf{1}}$$

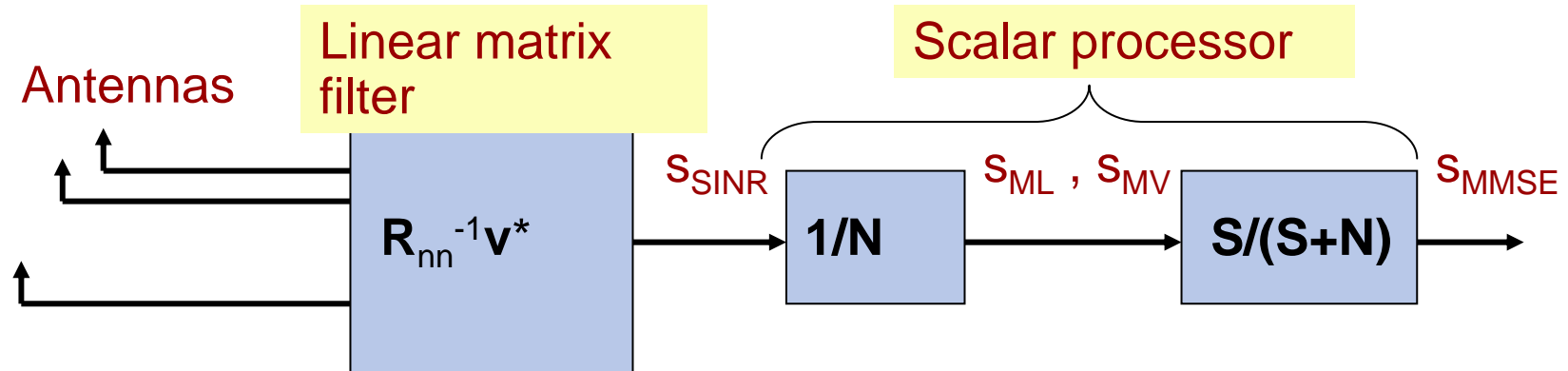
S = signal power, v^* = array response vector of the desired signal, R_{nn} = interference+noise covariance matrix, R_{xx} = total received signal (signal+interference+noise) covariance matrix, $v = \mathbf{1}$ for known signal weights in MV

Note:

here $\mathbf{v} = \mathbf{r}_{xd}$

$\mathbf{x}(t) = s(t)\mathbf{v} + \mathbf{n}(t)$

Optimal combining weights



- Notations used above:

$$\mathbf{x}(t) = s(t)\mathbf{v} + \mathbf{n}(t)$$

$$\mathbf{R}_{xx} = \mathbf{E}\{\mathbf{x}(t) \mathbf{x}^*(t)\}$$

$$\mathbf{R}_{nn} = \mathbf{E}\{\mathbf{n}(t) \mathbf{n}^*(t)\}$$

$$\mathbf{R}_{ss} = \mathbf{E}\{s(t) s^*(t)\}$$

$$\mathbf{s}(t) = s(t)\mathbf{v}$$

$$\mathbf{R}_{xx} = \mathbf{R}_{ss} + \mathbf{R}_{nn}$$

Note:

$\mathbf{n}(t)$ includes both interference and AWGN:
 $\mathbf{n}(t) = \mathbf{l}(t) + \mathbf{N}_0$

\mathbf{v} is the array response vector corresponding to desired signal direction

Adaptive antenna algorithms

- Adaptive antenna algorithms solve the optimal antenna weights for (rapidly) changing radio channel state
- Different criteria (MMSE, max SINR, ML, MV) lead to the same solution

$$\mathbf{w}_{\text{opt}} = \alpha \mathbf{R}_{\text{nn}}^{-1} \mathbf{r}_{\text{xd}}^*$$

- Task is to estimate \mathbf{R}_{nn} and \mathbf{r}_{xd} and find a simple and robust (direct or iterative) way to invert \mathbf{R}_{nn}
- \mathbf{R}_{nn} characteristics have an important role in the adaptation

For more details about matrix inversion, see

S. Haykin: "Adaptive filter theory",
Prentice Hall

- Typical adaptive antenna algorithms:

- *LMS*
- *DMI*
- *RLS*

Example: DMI algorithm

- Direct matrix inversion

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xd}^*$$

- only estimates of \mathbf{R}_{nn} and \mathbf{r}_{xd} are known:

$$\mathbf{E}\{\mathbf{R}_{xx}\} = \hat{\mathbf{R}}_{xx} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k) \mathbf{x}^*(k)$$

$$\mathbf{E}\{\mathbf{r}_{xd}\} = \hat{\mathbf{r}}_{xd} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k) d^*(k)$$

- Thus we get

$$\hat{\mathbf{w}}_{opt} = \hat{\mathbf{R}}_{nn}^{-1} \hat{\mathbf{r}}_{xd}$$

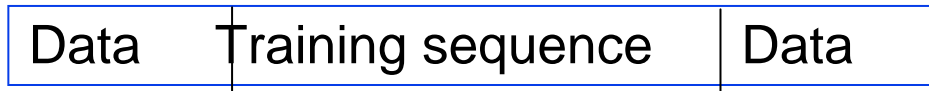
if \mathbf{R}_{nn} can be estimated ($\alpha = 1$)
or if only \mathbf{R}_{xx} can be estimated

$$\hat{\mathbf{w}}_{opt} = \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{r}}_{xd}$$

DMI algorithm

- Example: GSM burst

58 bit 26 bit (**d**) 58 bit



$$E\{\mathbf{R}_{xx}\} = \hat{\mathbf{R}}_{xx} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k)\mathbf{x}^*(k)$$

$$E\{\mathbf{r}_{xd}\} = \hat{\mathbf{r}}_{xd} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k)d^*(k)$$

- Procedure

- calculate $E\{\mathbf{R}_{xx}\}$ over the whole burst ($K=148$)
- calculate $E\{\mathbf{r}_{xd}\}$ over the known symbols ($K=26$)

(actually $E\{\mathbf{r}_{xd}\} = E\{\mathbf{h}\}$ = estimate of the impulse response)

$$\hat{\mathbf{w}}_{opt} = \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{r}}_{xd}$$

- $E\{\mathbf{R}_{xx}\}$ and $E\{\mathbf{r}_{xd}\}$ are calculated for each burst
- $E\{\mathbf{R}_{xx}\}$ and $E\{\mathbf{r}_{xd}\}$ can be averaged over several bursts
- If \mathbf{r}_{xd} is known and \mathbf{R}_{nn} can be estimated:

=> $K > 3M$ (roughly)

$$E\left\{\frac{snr}{snr_{opt}}\right\} \cong \frac{K + 2 - M}{K + 1}$$

Summary

- Simple beamforming (beamforming) is simple and robust and applies well to FDD systems
- Adaptive antenna algorithms can be applied to interference suppression
- Adaptive antenna algorithms/ optimum combining is required for good MIMO performance: "space-time equalisation"

References

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