Performance Bounds for Large Wireless Networks with Mobile Nodes and Multicast Traffic

Stavros Toumpis
Vienna Telecommunications
Research Center (ftw.)

Andrea J. Goldsmith
Wireless Systems Lab
Stanford University

IWWAN 2004

Outline

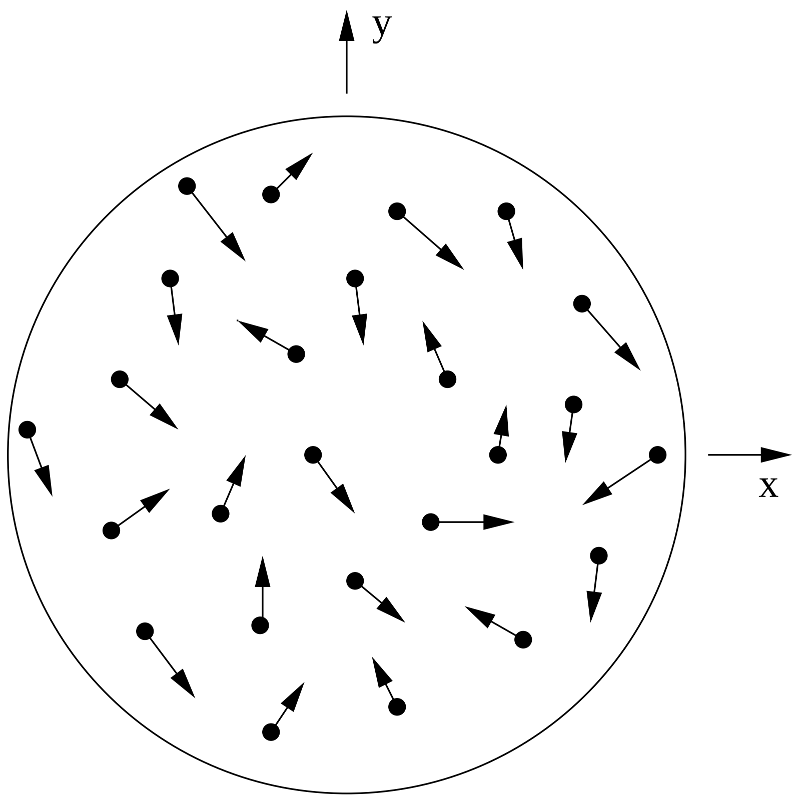
- We investigate the traffic-carrying capabilities of wireless ad hoc networks when:

  1. Nodes are mobile.
  2. There are packet delay constraints.
  3. The traffic is multicast, i.e., each packet has multiple destinations.
  4. The communication channel exhibits flat fading.

- Methodology introduced by P. Gupta and P. R. Kumar in "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

  – We develop communication schemes that will work with probability going to 1 as the number of nodes goes to infinity.
  – But the intuition we develop also applies to networks with a moderate number of nodes.

Networks with Mobile Nodes (Grossglauser and Tse)



- n mobile nodes $X_1, \ldots, X_n$, moving randomly and independently in a disk.
- Each node has a random destination node, all nodes require common end-to-end rate $\lambda(n)$ bps.
- A transmission from node X_i to node X_j is successful if and only if the Signal to Interference and Noise Ratio (SINR) exceeds a global value γ.
- All transmissions are with rate W.

Mobility Increases the Capacity (Grossglauser and Tse)

- **With high probability (w. h. p.)**, i.e., with probability going to 1 as the number of nodes n goes to infinity, each node is guaranteed an end-to-end rate k_1, which is not a function of n:

$$\boxed{\exists k_1 > 0 : \quad Pr\{\lambda(n) = k_1 \text{ is achievable}\} \rightarrow_{n\rightarrow\infty} 1.}$$

- A remarkable result! In networks with immobile nodes, the maximum $\lambda(n)$ necessarily decays approximately like $\frac{1}{\sqrt{n}}$.

- **The catch: delay increases very fast with n.**
  - Original paper gives no bounds.
  - Under reasonable assumptions, delay increases linearly with n.

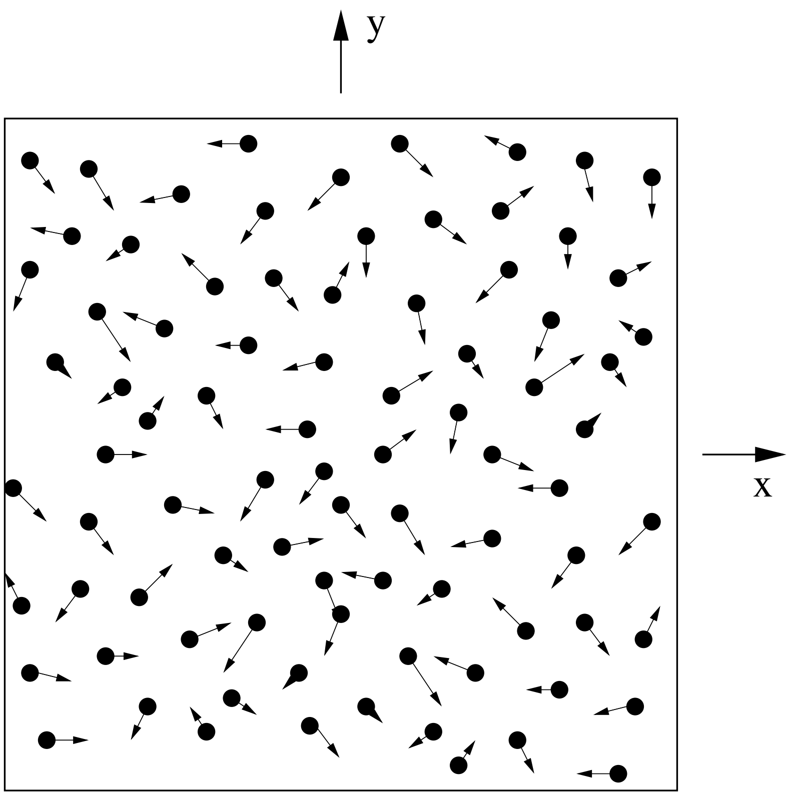
Our Motivation

- Grossglauser and Tse showed that it is possible to achieve **great throughput** but at the cost of **terrible delay**.
  - Can we exchange the two?

- Most work on the capacity of wireless networks assumes that traffic is unicast, i.e., each packet has a single destination.
  - But in many applications, traffic is multicast.
  - For example, consider a group of soldiers, each equipped with a transceiver, forming a wireless ad hoc network (the archetypal application).

- Fading has been ignored, but its effects may be dramatic.

Mobility Model



- n nodes placed in square area, and their movements are independent and uniform.

- For simplicity:
  - Nodes are perfectly reshuffled every S secs.
  - Within B secs, nodes do not move.

- But results also hold for Brownian motion, various random walks, etc.

- Experiment lasts for $2Nn^D$ frames of duration B, where D is integer, greater than 1. No node is moving for the duration of a frame.

Propagation Model

- When node Z_i transmits with power P_i, node Z_j receives a signal with power $P_j^r = G_{ij} P_i$, where $G_{ij} = (K d_{ij}^{-\alpha} f_{ij})$, with $\alpha > 2$.

- f_{ij} models fading. Its distribution has a thin tail:

$$F^c(x) \triangleq P[f_{ij} > x] \leq \exp[-qx] \; \forall x > x_1.$$

- Reception in node Z_j will be successful provided the transmission rate R_j satisfies:

$$R_j \leq f_R(\gamma_j) \triangleq W \log_2(1 + \frac{1}{\Gamma}\gamma_j),$$

  where

$$\gamma_j = \frac{G_{ij} P_i}{\eta W + \sum_{k \in \mathcal{T},\ k \neq i} G_{kj} P_k}.$$

Traffic Model

- Each node creates data packets with a common rate $\lambda(n)$ bps.
- Each node must transmit its packets to $m(n)$ other nodes, chosen randomly and uniformly among the rest of the nodes. We assume that for sufficiently large n,

  $$k_2 n^a \leq m(n) \leq k_3 n^a,$$

  where $a \in (0,1)$ is the **multicast exponent**.

- We measure the aggregate throughput of a communication scheme $T(n)$ *at the destinations*:

  $$T(n) \triangleq \lambda(n) \times n \times m(n).$$

Overview of Results

- The (very simple) **time division scheme** achieves:
  - An aggregate throughput, measured at the destinations, equal to $T(n) = n^{a-\epsilon}$, for any $\epsilon > 0$.
  - A maximum packet delay $d_{\max} = B$.

- The (more complicated) **tradeoff scheme** achieves:
  - An aggregate throughput, measured at the destinations, equal to $T(n) = n^{\frac{1+d}{2}-\epsilon}$, for any $\epsilon > 0$, where $d \in (0,1)$ is a design parameter.
  - A maximum packet delay $d_{max}(n) = k_4 n^d$.

- Both results hold with high probability, i.e., with probability going to 1 as the number of nodes goes to infinity. Furthermore, the convergence is *exponentially fast*.

Time Division Scheme - No Fading

- Assume that all fading coefficients are $f_{ij} \triangleq 1$.
- Let there be only one node transmitting at any given time.
- Nodes take turns transmitting.
- The minimum SNR at any receiver is $\frac{KP}{\eta W D^\alpha}$, where D is the diameter of the area.
- The following aggregate throughput is achievable:

$$T(n) = m(n) \times W \log_2(1 + \frac{KP}{\Gamma \eta W d_{\max}^\alpha}).$$

- A catch: what if the minimum SNR is very, very small?

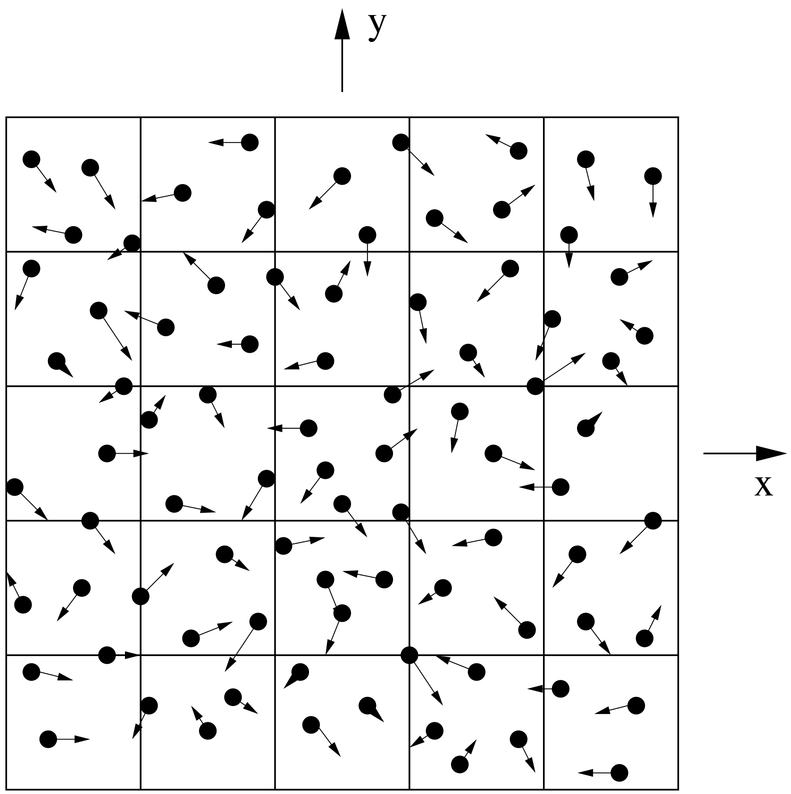
Time Division Scheme - Fading

- In the presence of fading, we can not lower bound the SINR of a transmission.

- Solution:
  - Packets are transmitted around n^ϵ times, each time by a different transmitter, for an arbitrary $\epsilon > 0$.
  - Transmissions are with rate $\frac{k_5}{\log n}$.
  - Whenever the transmission coefficient f_{ij} between two nodes is greater than $\frac{1}{2}$, a transmission between them will be successful.
  - Provably, all n^a destinations will receive the packet successfully, in one of the n^ϵ transmissions.

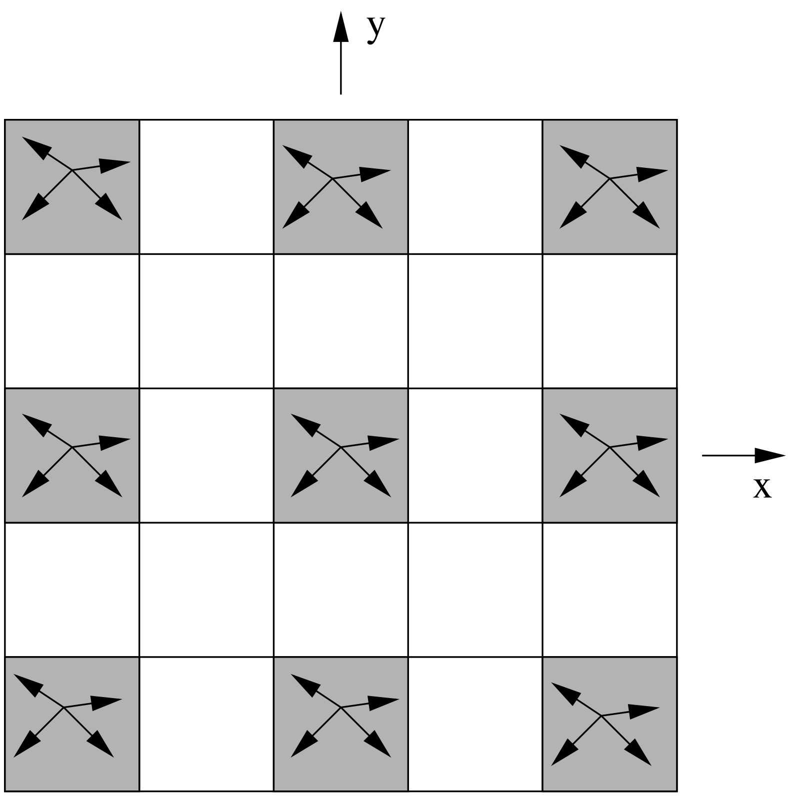
- The throughput is reduced by $n^\epsilon \log n$ with respect to the non-fading case.

Tradeoff Scheme - Cell Lattice



- n nodes.
- $n^{\frac{d+1}{2}-\epsilon}$ cells $C_1, C_2, \ldots$
- $d \in (0,1)$ is a design parameter, and $\epsilon > 0$.
- Let m_{ij} be the number of nodes in cell C_i, in frame j. Then $E[m_{ij}] = n^{\frac{d-1}{2}+\epsilon}$.
- **Lemma:** With high probability, $\forall i, j,\ \frac{1}{2}E[m_{ij}] < m_{ij} < 2E[m_{ij}]$.
- **Proof:** Chernoff bounds, Union bound.

Tradeoff Scheme - Spatial Reuse



- Divide cells in 4 regular sub-lattices.
- Divide each frame in 4 slots of duration $\frac{B}{4}$.
- In each slot, only one node is allowed to transmit, in each of the cells of the corresponding sub-lattice. The rest of the nodes in the cell will try to receive the packet.
- **Lemma:** With high probability, the SINRs of at least 50% of the links at all slots are greater than $\frac{1}{\log n}$.
- **Proof:** Same as in previous cases.

Packet Transmissions

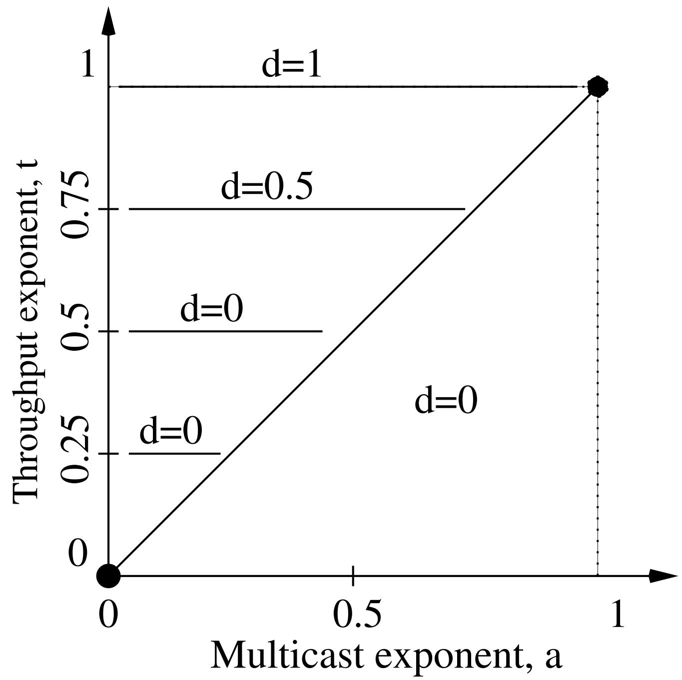
- Odd frames: Source-Relay Communication.
  - Each node transmits a single packet, intended for its $m(n)$ destinations.
  - Nodes that receive it will act as relays in subsequent **even** frames.
  - Data rate used: $R(n) = \frac{1}{\log n}$.
  - Packet duration: $D(n) = n^{\frac{d-1}{2} - 2\epsilon}$.
  - W. h. p., all nodes will get their chance to transmit a packet.
- Even frames: Relay-Destination Communication.
  - Packets in the same cell with their destination are delivered.
  - W. h. p., all packets will have the time to be transmitted.
- Basic tradeoff: the larger the cells are, the more relays each packet has (and the smaller the packet delay will be), but the smaller the number of simultaneous transmissions can be (and the smaller the throughput becomes).

Throughput and Delay Calculation

- Throughput calculation:
  - Every $2B$ seconds, each node creates a packet.
  - Each packet has a size of $R(n) \times D(n)$ bits.
  - Aggregate throughput is $T(n) = \frac{R(n) \times D(n)}{2B} = k_5 n^{\frac{d+1}{2} - 3\epsilon}$.

- Delay calculation:
  - Each message is carried by around $r(n) = n^{\frac{1-d}{2} + \epsilon}$ relays.
  - These relays spread out in $c(n) = n^{\frac{1+d}{2} - \epsilon}$ cells.
  - Probability that a packet will make it in a frame is only $\frac{r(n)}{c(n)} << 1$.
  - We need around $\frac{c(n)}{r(n)}$ frames, or $2S\frac{c(n)}{r(n)}$ seconds.
  - **Lemma:** W. h. p., all packets delivered with a delay smaller than $d_{\max} = k_4 n^d$.

Conclusions



- Number of destinations per packet $m(n) \sim n^a$.
- Aggregate throughput $T(n) \sim n^t$.
- Packet delay $d(n) \sim n^d$.