Analysis of Amplify-and-Forward DSTBCs over the Random Set Relay Channel

Giuseppe Abreu
Qiang Xue

http://www.cwc.oulu.fi/~giuseppe

Centre for Wireless Communications
University of Oulu

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DSTBC over Uncoordinated Relay Pool

- Decode-and-forward DSTBC [Laneman’03, Barbarossa’04]
- Effect of node distribution on DF-DSTBC [Sadek’05]
- Best-relay AF [Bletsas’07, Krikidis’08]


AF-DSTBC over Uncoordinated Relay Pool

- $P_1$: Transmit power of the source
- $f_k$: Source-relay channel coefficient (Rayleigh, $E[|f_k|^2] = 1$)
- $\gamma_{ar_k}$: Instantaneous SNR of source-to-$r_k$ channel
- $\sigma_1^2$: Average noise power at each relay

AF-DSTBC over Uncoordinated Relay Pool

- $P_2$ : Transmit power of each relay (constant)
- $g_k$ : Source-relay channel coefficient (Rayleigh, $E[|g_k|^2] = 1$)
- $\gamma_{r_kb}$ : Instantaneous SNR of $r_k$-to-destination channel
- $\sigma^2_2$ : Average noise power at destination

The Random Set Relay Channel

- Relay active iff $|f_k|^2 \geq \xi$

- Relay channel: $\mathcal{C} \triangleq \{f_1g_1, \cdots, f_Kg_K\}$ (random set)

$$p_K(\xi, N) = \binom{N}{K} e^{-K\xi}(1 - e^{-\xi})^{(N-K)}$$

$$p_K(\xi, \nu) = \lim_{N \to \infty} p_K(\xi, N) = \frac{\nu^K e^{-\nu}}{K!}$$

- Assumption: $f_k$’s are i.i.d

- Using: $\Pr\{|f_k|^2 \geq \xi\} = e^{-\xi}$
Amplification Strategies

- With CSI

\[ \rho = \sqrt{\frac{P_2}{(1 + \xi)P_1 + \sigma_1^2}} \cdot \frac{f}{|f|} \]

- Without CSI [Maham’08]

\[ \rho = \sqrt{\frac{P_2}{(1 + \xi)P_1 + \sigma_1^2}} \]

- With linear DSTBC, achieves the same performance

Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

\[ \gamma_{ab} = \frac{\rho^2 P_1 \sum_{k=1}^{K} |f_k|^2 |g_k|^2}{\sigma_2^2 + \rho^2 \sigma_1^2 \sum_{k=1}^{K} |g_k|^2} \]
Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

\[
\gamma_{ab} = \alpha \sum_{k=1}^{K} |f_k|^2 |g_k|^2
\]

\[
\alpha = \frac{\bar{\gamma}_{ar} \bar{\gamma}_{rb}}{1 + (1 + \xi) \bar{\gamma}_{ar} + \bar{\gamma}_{rb} \sum_{k=1}^{K} |g_k|^2}
\]
Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

\[ \gamma_{ab} = \alpha \sum_{k=1}^{K} |f_k|^2 |g_k|^2 \]

\[ \alpha \approx \frac{\tilde{\gamma}_{ar} \tilde{\gamma}_{rb}}{1 + (1 + \xi) \tilde{\gamma}_{ar} + K \tilde{\gamma}_{rb}} \]

\[ K \gg 1 \implies \sum_{k=1}^{K} |g_k|^2 \approx K \]
Statistics of the RSRC

- Define: \( z = |f|^2 \cdot |g|^2 \), \( x \triangleq |f|^2 \) and \( y \triangleq |g|^2 \)
- Recall: \( p_X(x) = e^{-x} \) and \( p_Y(y) = e^{-y} \)
- Then: \( F_Z(z|x \geq \xi) = \int_\xi^\infty \Pr\{y \leq z/x\} \cdot p_X(x \geq \xi) \, dx \)
  \[ = \int_\xi^\infty \left(1 - e^{-z/x}\right) \cdot e^{\xi-x} \, dx \]
  \[ = 1 - e^{\xi} 2\sqrt{z} K_1(2\sqrt{z}) + e^{\xi} \int_0^\xi e^{-\frac{z}{x} - x} \, dx \]
  \[ p_Z(z|x \geq \xi) = \frac{d}{dz} F_Z(z|x \geq \xi) \]
- Thus: \( p_Z(z|x \geq \xi) = 2e^{\xi} K_0(2\sqrt{z}) - e^{\xi} \int_0^\xi \frac{1}{x} e^{-\frac{z}{x} - x} \, dx \)
- And: \( \mu_z(-s; \xi) = \frac{1}{s} e^{\xi} + \frac{1}{s} E_1(\xi + 1/s) \)
BER of Regular AF-DSTBC over the RSRC

- General expression

\[
\bar{P}_X(\bar{\gamma}_{ar}, \bar{\gamma}_{rb}; \xi, \eta, N, M) = \sum_{K=0}^{\frac{N}{K}} p_K(\xi, N) \cdot \bar{P}_X(\bar{\gamma}_{ar}, \bar{\gamma}_{rb}; \xi, \eta K, M)
\]

- With PSK:

\[
\bar{P}_{PSK} = \sum_{m=1}^{M-1} \frac{\bar{d}_{m:PSK}}{2 \log_2 M} \cdot \left[ I(\delta_m^-; \alpha \cdot \Delta_{PSK}(\delta_m^-), \eta K) - I(\delta_m^+; \alpha \cdot \Delta_{PSK}(\delta_m^+), \eta K) \right]
\]

- With QAM:

\[
\bar{P}_{QAM} = \sum_{m=1}^{\log_2 \sqrt{M}} \left( (1 - 2^{-m}) \sqrt{M} - 1 \right) \frac{4d_{i:QAM}}{\sqrt{M \log_2 \sqrt{M}}} \cdot I \left( \frac{1}{2}, \alpha \cdot \Delta_{QAM}(i), \eta K \right)
\]

- Where: \( I(\delta, \alpha \cdot \Delta, \eta K) = \frac{1}{\pi} \cdot \int_0^{(1-\delta)} \left[ \mu_z \left( -\frac{\alpha \cdot \Delta}{\sin^2(\theta); \xi} \right) \right]^{\eta K} d\theta \),

Diversity of Regular AF-DSTBC over the RSRC

• Definition [Jafarkhani’05]:

\[
G(K) \triangleq - \lim_{\bar{\gamma} \to \infty} \frac{\log (\text{SER}(K))}{\log(\bar{\gamma})}
\]

• Symbol Error Rate

\[
\text{SER}(K) = c \cdot Q\left(\sqrt{\alpha \cdot \Delta \sum_{k=1}^{K} |f_k \cdot g_k|^2}\right) \leq \frac{c}{2} \mu_z^{\eta K} \left(- \frac{\alpha \cdot \Delta}{2}; \xi\right)
\]

• Diversity gain

\[
G(K) = - \lim_{\bar{\gamma} \to \infty} \frac{\eta K \left(- \log (\chi) + \frac{1}{\chi} + \log[\log(\chi)]\right)}{\log(\bar{\gamma})}
\]

\[
= \lim_{\bar{\gamma} \to \infty} \frac{\eta K \log(\chi)}{\log(\bar{\gamma})} = \eta K, \quad \chi \propto \bar{\gamma}
\]

- Use \( E_1(x) = \varepsilon + \log(-x) + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} \)

Performance of Ideal AF-DSTBC in RSRC

Relays Close to Destination

Analytical Performance of Ideal AF-DSTBC x Threshold

\( (\eta = 1, N = 8, \text{QPSK}, \gamma_{rb} \to \infty) \)

BER

\( \xi \)

\( \gamma_{ar} = 0\,\text{dB} \)

\( \gamma_{ar} = 5\,\text{dB} \)

\( \gamma_{ar} = 10\,\text{dB} \)

\( \gamma_{ar} = 15\,\text{dB} \)

\( \gamma_{ar} = 20\,\text{dB} \)

Optimal \( \xi \)
Performance of Ideal AF-DSTBC in RSRC
Relays in Midrange

Analytical Performance of Ideal AF-DSTBC x Threshold
($\eta = 1, N = 8, \text{QPSK}, \gamma_{rb} = 10\text{dB}$)

BER
$
\xi
$
$
\bar{\gamma}_{ar}
$
0 dB
5 dB
10 dB
15 dB
20 dB
Optimal $\xi$

κ =3.5
κ =4.5
κ =5.8
κ =7.1
κ =7.8
Performance of AF-GABBA with Genie Relays in Midrange

Simulated Performance of AF-DGABBA x Threshold
\( (N = 8, \text{QPSK}, \bar{\gamma}_{rb} = 10\text{dB}) \)

BER
\( \bar{\gamma}_{ar} = 0\text{dB} \)
\( \bar{\gamma}_{ar} = 5\text{dB} \)
\( \bar{\gamma}_{ar} = 10\text{dB} \)
\( \bar{\gamma}_{ar} = 15\text{dB} \)
\( \bar{\gamma}_{ar} = 20\text{dB} \)
Optimal \( \xi \)
Analytical Performances of Ideal AF-STBC w/wo Autonomic Relay Selection

\( (N = 8, 16QAM, \bar{\gamma}_{rb} = 10dB) \)

- BER
  - \( \kappa = 7.2 \), \( \xi = 0.0 \)
  - \( \kappa = 5.9 \), \( \xi = 0.2 \)
  - \( \kappa = 4.9 \), \( \xi = 0.4 \)
  - \( \kappa = 4.0 \), \( \xi = 0.6 \)
  - \( \kappa = 3.3 \), \( \xi = 0.8 \)
  - \( \kappa = 2.7 \), \( \xi = 1.0 \)
  - \( \kappa = 2.2 \), \( \xi = 1.2 \)
  - \( \kappa = 1.8 \), \( \xi = 1.4 \)
  - \( \kappa = 1.5 \), \( \xi = 1.6 \)
  - \( \kappa = 1.2 \), \( \xi = 1.8 \)
  - \( \kappa = 1.0 \), \( \xi = 2.0 \)
Spectral Efficiency of Ideal AF-DSTBC in RSRC
Small Pool

Analytical Performances of Ideal AF-STBC w/wo Autonomic Relay Selection
(N = 8, 16QAM, $\bar{\gamma}_{rb} = 10$dB)
Spectral Efficiency of Ideal AF-DSTBC in RSRC Large Pool

Analytical Performances of Ideal AF-STBC w/wo Autonomic Relay Selection
(N = 20, 16QAM, $\bar{\gamma}_{rb} = 10$dB)

<table>
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<th>$\xi$</th>
<th>$\kappa$</th>
</tr>
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<tbody>
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<td>0.0</td>
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<tr>
<td>0.2</td>
<td>14.8</td>
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<td>0.4</td>
<td>12.1</td>
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<td>9.9</td>
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<tr>
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<tr>
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<tr>
<td>1.8</td>
<td>3.0</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Spectral Efficiency of Ideal AF-DSTBC in RSRC Large Pool

Analytical Performances of Ideal AF-STBC w/wo Autonomic Relay Selection
\( (N = 20, 16\text{QAM}, \bar{\gamma}_{rb} = 10\text{dB}) \)
Average Mutual Information of AF-DSTBC

• Instantaneous SNR (without forward CSI):

\[
\gamma_{ab} = \frac{\rho^2 P_1 \sum_{k=1}^{K} |f_k|^2 |g_k|^2}{\sigma^2 + \rho^2 \sigma^2 \sum_{k=1}^{K} |g_k|^2}
\]

• Mutual Information (for a given \( \{f_1 g_1, \cdots, f_K g_K\} \in \mathcal{C}\)):

\[
I = \frac{1}{2} \log(1 + \gamma_{ab})
\]

- Where \( \log \) is in base 2

• Average Mutual Information:

\[
\bar{I} = p_1(\xi, N) \cdot \bar{I}_{K=1} + p_2(\xi, N) \cdot \bar{I}_{K=2} + \cdots + p_N(\xi, N) \cdot \bar{I}_{K=N}
\]
Average Mutual Information: \((K = 1)\)

- **Instantaneous SNR (conditioned on \(K = 1\)):**

\[
\gamma_{ab}\big|_{K=1} = \frac{\rho^2 P_1 |f|^2 |g|^2}{\sigma^2 + \rho^2 |g|^2}
\]

- **Average Mutual Information:**

\[
\bar{I}_{K=1} = \frac{1}{4 \ln 2} \int_0^\infty \int_0^\infty \ln \left( 1 + \frac{\bar{\gamma}_{ar}(x + \xi) y}{y + \varepsilon} \right) e^{-x} e^{-y} \, dx \, dy
\]

\[
= \frac{1}{4 \ln 2} \left[ e^\varepsilon \text{Ei}(\varepsilon) - e^{\bar{\gamma}_{ar} \xi + 1} \text{Ei}\left( \frac{-\varepsilon}{\bar{\gamma}_{ar} \xi + 1} \right) - e^{-\bar{\gamma}_{ar}} \int_0^\infty \text{Ei}\left( \frac{-\varepsilon - (\bar{\gamma}_{ar} \xi + 1) y}{\bar{\gamma}_{ar} y} \right) e^{\frac{\varepsilon - \bar{\gamma}_{ar} y^2}{\bar{\gamma}_{ar} y}} \, dy \right]
\]

- **Where:** \(\varepsilon \triangleq 1/\rho^2 = \frac{(1+\xi)P_1+\sigma^2}{P_2}\)
Bounds on $\tilde{I}_{K=1}$

- **Upper Bound:**

  \[
  \mathbb{E}[\log(1 + x)] \leq \log(1 + \mathbb{E}[x]) \Rightarrow \mathbb{E}[\log(1 + \gamma_{ab})] \leq \log \left( 1 + \frac{\gamma_{ab} K (1 + \xi)}{K + \varepsilon} \right)
  \]

  \[
  \tilde{I}_{K=1} \leq \frac{1}{4} \log \left( 1 + \frac{\gamma_{ar} (1 + \xi)}{1 + \varepsilon} \right)
  \]

- **Lower Bound:**

  \[
  \log(2\sqrt{x}) < \log(1 + x)
  \]

  \[
  \tilde{I}_{K=1} \geq \ln 2 + \frac{1}{2} \left[ \ln \frac{\gamma_{ar} \xi}{\varepsilon} + e^\varepsilon \text{Ei}(-\varepsilon) - e^\xi \text{Ei}(-\xi) - \mathcal{Z} \right]
  \]

  - Where: $\mathcal{Z}$ is the Euler constant
Conditional Average Mutual Information: \((K \gg 1)\)

\[
K \gg 1 \iff \sum_{k=1}^{K} |g_k|^2 \approx K
\]

- **Instantaneous SNR (conditioned on \(K \gg 1\)):**

\[
\gamma_{ab} \bigg|_{K \gg 1} = \frac{P_1}{\sigma^2} \frac{K(1 + \xi)}{(1 + \xi)P_1 + \sigma^2} + K
\]

- **Average Mutual Information:**

\[
\bar{I}_{K \gg 1} = \frac{1}{4} \log \left(1 + \frac{\tilde{\gamma}_{ar} K(1 + \xi)}{K + \varepsilon}\right)
\]
Conditional Average Mutual Information: \((K = 1)\)

Average Mutual Information × Threshold

\(K = 1, \bar{\gamma}_{ar} = \bar{\gamma}_{rb} = 10\text{dB}\)

- **Upper Bound**
- **Lower Bound**
- **Exact**
Conditional Average Mutual Information: \((K = 2)\)

Average Mutual Information \times \text{Threshold} \\
\((K = 2, \bar{\gamma}_{ar} = \bar{\gamma}_{rb} = 10\text{dB})\)

- Upper Bound
- Lower Bound
Average Mutual Information (Fixed $N$)

Average Mutual Information $\times$ Threshold
(Various $N, \bar{\gamma}_{ar} = \bar{\gamma}_{rb} = 10\text{dBm}$)

bits/sec $\times$ Hz

$\xi$

$N = 8$
$N = 12$
$N = 16$
Maximum Average Mutual Information

Maximum Average Mutual Information $\times$ Threshold
($\bar{\gamma}_{ar} = \bar{\gamma}_{rb} = 10\text{dBm}$)

0 10 20 30 40 50 60 70 80 90 100

Analytical

$\frac{5.5}{\sqrt{\ln(1 + N/10)}}$
Maximum Average Mutual Information

Average Mutual Information $\times$ Threshold

$(\gamma_{ar} + K \cdot \gamma_{rb} = 13\text{dB})$

Average Mutual Information $\times$ Threshold

$\beta = \bar{\gamma}_{ar}/\bar{\gamma}_{ab}$

$\xi = 0 : 0.1 : 1.3$
Maximum Average Mutual Information

Average Mutual Information × Threshold
($\gamma_{ar} + K \cdot \gamma_{rb} = 13\text{dB}$)

$\beta = \bar{\gamma}_{ar}/\bar{\gamma}_{ab}$

$\xi = 1.4 : 0.1 : 2.2$
Conclusions

Opportunistic Cooperation Works!
(Better BER, Efficiency and Mutual Information)