Downlink-Assisted Uplink Zero-Forcing for TDD Multiuser MIMO Systems

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Introduction to Multiuser MIMO

- Multiple antennas in base station and mobiles
- Multiple spatial beams (modes) per user allowed
- Rich scattering environment, non-line-of-sight
- Geographic direction not obvious
TDD system: Assumptions

- Uplink and downlink channel reciprocity
  - UL and DL frames on same frequency band
  - In OFDMA, same subcarriers used for UL and DL
  - TX uses the knowledge of CSI obtained during RX

- Space division multiple access (SDMA)
  - A number of users co-exist in the same time/frequency frame
  - Beam/user selection (scheduling)

- Opportunistic rate allocation
  - Constant (sum) TX power with per frame variable rates

- Relatively small cells and low mobility
  - Simultaneous users are symbol-synchronous
  - No inter-symbol-interference (cyclic prefix)
TDD system: Signal model

DL and UL RX signals:

Channel

\[ \mathbf{x}_k^d(n) = \sum_{i=1}^{K} \mathbf{H}_k \mathbf{M}_i^d \mathbf{A}_i^d \mathbf{b}_i^d(n) + \mathbf{v}_k^d(n) \]

Data

\[ \mathbf{x}_k^u(n) = \sum_{i=1}^{K} \mathbf{H}_i^H \mathbf{M}_i^u \mathbf{A}_i^u \mathbf{b}_i^u(n) + \mathbf{v}_k^u(n) \]

\( \mathbf{H}_k \in \mathbb{C}^{N_u \times N_b} \)

\( \mathbf{M}_k^d \in \mathbb{C}^{N_b \times L_k} \)

\( \mathbf{M}_k^u \in \mathbb{C}^{N_u \times L_k} \)
Greedy beam selection

- Greedy [4,5] user/beam selection by BS
- The same set of beams are active in both directions
  - Beams spatially compatible in DL are compatible also in UL
- Selection is based on user-specific eigenmodes
  - Select consecutively, one at a time
  - At each step, select eigenbeam with largest component orthogonal to previously selected beams
  - Stop selecting if calculated capacity reduces or when all DoF’s are used

- Beamforming gain when SNR low
- Multiplexing gain when SNR high
Tx-Rx UL-DL zero-forcing

- No restrictions for the number of antennas in active terminals [1]
- UL is based on reversal of DL signal processing chain
- DL receiver orthonormalized into UL precoder
- Tx power + rate allocation, and scheduling/beam selection decoupled from Tx precoder design

ZF criterion:

\[
F_k^H H_k C_i = 0, \quad i \neq k
\]
Iterative ZF solution

\[ H_k = U_k \Lambda_k V_k^H \]

Algorithm 1 Iterative zero-forcing

1) Initialize \( F_k \) for each user with the chosen \( L_k \) left singular vectors from matrix \( U_k \).
2) Find \( C_k \) for each user so that \( F_k^H H_k C_i = 0, \ i \neq k \).
3) Calculate SVD: \( H_k C_k = \tilde{U}_k \Lambda_k \tilde{V}_k^H \) and set \( F_k \) to contain the \( L_k \) first column vectors from \( \tilde{U}_k \).
4) If \( F_k^H H_k C_i \approx 0, \ i \neq k \) with sufficient accuracy, stop iteration. Otherwise repeat steps 2 to 4.
5) Select transmit precoders as \( M_k^d = C_k \tilde{V}_k \).

- Iteration starts from user-specific eigenmodes [3]
- Ideally DL receiver is matched filter; active interference suppression not mandatory => single-user processing by terminals
- In practice multi-user interference exists in DL due to imperfect CSI => interference suppression is beneficial
Pilot signals

- The same precoded pilots used for the purposes of Rx and Tx
- In DL, precoded pilots associated with the active data beams are sufficient
- In UL, append the pilot precoder matrix to be unitary
  => UL pilot overhead grows with the aggregate number of user antennas (as with all MU-MIMO schemes)

\[
\tilde{M}_k^u = \begin{bmatrix} M_k^u & \tilde{M}_k^u \end{bmatrix} \in \mathbb{C}^{N_u \times N_u}
\]

- pilot precoder matrix
- data precoders
- additional precoders
Pilot signals: Rx responses

- Pilot responses are the only reference for spatial multiuser processing

\[
\mathbf{R}^{d}_{k,i} = \mathbf{H}^{d}_{k} \mathbf{M}^{d}_{i} \in \mathbb{C}^{N_{u} \times L_{i}}
\]
\[
\mathbf{R}^{u}_{k} = \mathbf{H}^{H}_{k} \mathbf{M}^{u}_{k} \in \mathbb{C}^{N_{b} \times L_{k}}
\]
\[
\tilde{\mathbf{R}}^{u}_{k} = \mathbf{H}^{H}_{k} \mathbf{M}^{u}_{k} \in \mathbb{C}^{N_{b} \times N_{u}}
\]

- Modified equivalent ZF criterion becomes

\[
\mathbf{F}^{H}_{k} \tilde{\mathbf{R}}^{uH}_{k} \mathbf{C}_{i} = \mathbf{F}^{H}_{k} \mathbf{H}^{H}_{k} \mathbf{M}^{uH}_{k} \mathbf{H}_{k} \mathbf{C}_{i} = 0, \quad i \neq k
\]

unitary
Summary of UL-DL procedure

**User Terminal (UL TX):**
- Estimate DL pilot RX responses ($L_k$ pilot streams per user)
- SVD for the personal $N_u \times L_k$ channel or receiver
- Waterfill (WF) over each user’s $L_k$ eigenmodes for data
- UL pilots TX via $L_k$ data precoders and additional $N_u - L_k$ pilot precoders with equal powers => unitary pilot precoder matrix

**Base Node (DL TX):**
- Estimate all UL pilot RX responses ($N_u$ pilot streams per user)
- Perform beam selection and iterative multiuser ZF processing
- Waterfill (WF) over the chosen beams => Transmit $L_k$ data streams per user
- DL pilots TX via data precoders but with equal powers

$k=3$  $L_1 = 1$

$N_b = 4$  $L_2 = 0$

$N_u = 2$  $L_3 = 2$
Numerical results: Simulation setup

- Frequency flat fading, independent between antennas
- Waterfilling power allocation, assuming knowledge of $N_0$
- Common UL-DL power allocation rule: $P_k = \frac{P \cdot L_k}{\sum_i L_i}$

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Numerical results: Achievable rate

- The achievable data rate per stream $s$ for user $k$ can be evaluated from the SINR seen after transmitter-channel-receiver chain:

\[
\gamma_{k,s}^d = \frac{p_{k,s}^d |w_{k,s}^d H_k m_{k,s}^d|^2}{\sum_{(i,j) \neq (k,s)} p_{i,j}^d |w_{k,s}^d H_k m_{i,j}^d|^2 + N_0 |w_{k,s}^d|^2}
\]

\[
\gamma_{k,s}^n = \frac{p_{k,s}^n |w_{k,s}^n H_k m_{k,s}^n|^2}{\sum_{(i,j) \neq (k,s)} p_{i,j}^n |w_{k,s}^n H_i m_{i,j}^n|^2 + N_0 |w_{k,s}^n|^2}
\]

\[
R_{k,s} = \log_2 (1 + \gamma_{k,s}) \quad \text{bits/s/Hz}
\]
Performance: Ideal CSI

$$\text{SNR} = \frac{\sum P_k}{N_0}$$

- $N_b = 4$, $N_u = 2$, greedy beam selection
- Ideally UL and DL are equal in performance, given same power constraints
Performance: Estimated CSI and ZF RX

- \( \{K=4, N_b=4, N_u=2\} \), noisy CSI estimated from pilot only
- *Estimated CSI in RX* vs. *Estimated CSI in RX and TX*
- Only ZF processing used here
  - Zero-forcing RX in BS
  - Least-norm RX in UE (is ideally ZF when precoding was perfect)
- UL RX suffers from UL pilot power distribution

\[
\text{SNR}_{\text{pilot}} = \frac{N_{\text{pilot}} \sum_k P_k}{N_0}
\]
Performance: Estimated CSI and LMMSE RX

- \( \{K=4, N_b=4, N_u=2\} \), noisy CSI estimated from pilot only
- Estimated CSI in RX vs. Estimated CSI in RX and TX
- LMMSE RX used here
  - Only non-precoded UL seems to really benefit

\[
\text{SNR}_{\text{pilot}} = \frac{N_{\text{pilot}} \sum_{k} P_k}{N_0}
\]
Conclusion

- **Low overhead**
  - Co-existence of UL and DL taken into account
  - Single set of pilots sufficient for both coherent RX and TX precoding

- **Simplicity**
  - Decoupled MIMO channels accommodate simple power/rate allocation, as well as coding and modulation
  - Ideally, terminals perform single-user MIMO
  - Linear TX and RX processing

- **Performance**
  - Beamforming gain from TX CSI also in uplink
  - Opportunistically uses all available degrees of freedom
  - Zero-forcing means suboptimal use of MU-MIMO channel capacity
    - Loss is small due to user/beam selection
References


Questions?

Thank You!