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Part 1: Introduction
Interference in Wireless Networks
Interference Management

Wireless World

Interference management is critical
Interference Management in Cellular Networks: Historical Perspective
Single Isolated Cell

- Manhattan mobile phone system (1946)
- Interference managed by simply orthogonalizing users in time-frequency
Multiple Access Alternatives

TDMA

Time

Frequency

FDMA

Time

Frequency

CDMA

Time

Frequency

signal 1

signal 2

signal 3

signal 4

signal 5
Cellular Concept (Bell Labs)

Reuse spectrum geographically through cell splitting
Narrowband Cellular (AMPS, GSM)

- Bandwidth divided into narrowband channels (200 Khz in GSM), and users are assigned time slots (8 per channel in GSM)
- No in-cell interference (orthogonal users in cell)
- Interference across cells on same channel is minimized by reusing same channels only in cells far apart
Frequency Reuse

- Spectral efficiency of narrowband cellular is reduced by reuse factor
- Interference localized to narrow band
Wideband Cellular (CDMA)

- **Universal frequency reuse** – all users in all cells share same bandwidth (1.25 MHz in IS-95 standard)
- Interference from in-cell users can be controlled by using orthogonal codes (downlink) or successive interference cancellation (uplink)
- Interference from other cells is **averaged** and simply raises the noise floor

**Advantages:**
- No reduction in capacity due to reuse (no frequency planning needed)
- Graceful degradation in performance with users (soft capacity)
- Any technique that reduces power of interferers (soft handoff, voice activity detection) increases capacity
Disadvantages of CDMA

- In-cell interference reduces capacity (cannot be eliminated completely)
- Tight power control is needed to manage interference, and may be too expensive for data applications (with low duty cycle)
Wideband Cellular OFDM (LTE)

- Split up bandwidth into narrowband sub-channels, with every user having access to all sub-channels.
- Basic unit of resource is virtual channel (hopping sequence)? virtual channels are orthogonal in TF.

From Tse & Viswanath, Fundamentals of Wireless Comm.
Wideband Cellular OFDM

- **In-cell Orthogonalization**
  - Hopping sequences of users within cell are designed to be orthogonal
  - Users are assigned one or more virtual channels

- **Out-of-Cell Interference Averaging**
  - Hopping patterns in adjacent cells are chosen so that there is minimal overlap between any pairs
  - Interference from out-of-cell is averaged over band rather than being localized
Additional Resources for Interference Management
Multiple Antennas (MIMO)

- Multiple antennas provide diversity against fading and multiplexing gain for point-to-point links
- Can also be used for spatial separation of users – beamforming
- Additional degrees of freedom for interference management!
Basestation Cooperation

- Out-of-cell users become in-cell users
- Beamforming (downlink), joint decoding (uplink)
Relaying and User Cooperation

- Potential interferers become helpers (relays of information)
- Particularly useful in distributed interference management (ad hoc, mesh networks)
Dynamic Spectrum Access (Cognitive Radio)

- Primary and secondary users of spectrum
- Secondary users sense channels to determine presence/absence of primary users
- Probability of interfering with primary is constrained
Explosion in Wireless Data Traffic

Global Mobile Data Traffic Growth / Top-Line
Global Mobile Data Traffic will Increase 10-Fold from 2014–2019

How to accommodate exponential growth without new useful spectrum?
Through Improved PHY?

- Point-to-Point wireless technology mature
  - Modulation/demodulation
  - Synchronization
  - Coding/decoding (near Shannon limits)
  - MIMO

- Centralized (in-cell) multiuser wireless technology also mature
  - Orthogonalize users when possible
  - Otherwise use successive interference cancellation

Spectral efficiency gains from further improvements in PHY are limited!
By Adding More Basestations?
Through Improved Interference Management

Several useful engineering solutions for managing interference

But...

What are fundamental limits?
References

Part 2: Information Theory for Interference Channels
Claude Shannon’s Information Theory

Point-to-point Communication on AWGN channel

Capacity = $\log(1 + \text{SNR})$ bits/sec/Hz
Two User Gaussian Interference Channel

- Gaussian noise with unit variance; transmit power constraint $P$
- Capacity region?
- Known when $h \geq 1$
Strong Interference Setting: $h \geq 1$

- Both receivers decode both messages (compound MAC)
- Capacity region

\[
R_1 \leq \frac{1}{2} \log(1 + P) \\
R_2 \leq \frac{1}{2} \log(1 + P) \\
R_1 + R_2 \leq \frac{1}{2} \log(1 + P + h^2 P)
\]
Very Strong Interference Setting: \( h \geq \sqrt{1 + P} \)

- Each user first decodes interference treating intended input as noise; then subtracts interference to decode intended input.
- Capacity region

\[
R_1 \leq \frac{1}{2} \log(1 + P)
\]

\[
R_2 \leq \frac{1}{2} \log(1 + P)
\]

same as when interference is absent
Two User Gaussian Interference Channel

- Gaussian noise with unit variance; transmit power constraint $P$
- Capacity region?

Open problem when $h < 1$
What is Known?

- Simple schemes
  - Treat interference as noise
  - Orthogonalize users
  - Single user coding/decoding
- Sophisticated schemes
  - Exploit structure in interference
  - Joint coding/decoding
- Han-Kobayshi achievable scheme
  - Power splitting (common/private) and time-sharing
  - Best known inner bound to capacity region for two-user GIC
- Special case of H-K scheme achieves capacity to within 1 bit!
  - Etkin, Tse, Wang 2007
Treating interference as noise is optimal in low interference regime

![Graph showing the relationship between SNR and INR Threshold]
Main Result

Theorem

For the two user symmetric Gaussian interference channel satisfying

\[ h(1 + h^2 P) \leq 0.5 \]

treating interference as noise achieves the sum capacity, which is given by

\[ C_{sum} = \log \left[ 1 + \frac{P}{1 + h^2 P} \right] \]

Requires new outer bound!
Genie-aided Outer Bound

- Genie gives side-information to receivers

- Sum capacity of genie-aided channel is obvious outer bound to original channel
Properties of Genie: Usefulness

• **Useful:** if sum capacity of genie-aided channel (easily) derivable
  - Gaussian inputs are optimal
    \[ \implies \text{treating interference as noise is optimal} \]
  - \( C_{\text{sum}}(\text{genie-aided}) = I(X_{1G}; Y_{1G}, S_{1G}) + I(X_{2G}; Y_{2G}, S_{2G}) \)

• Eg: \( S_1 = X_2 \) and \( S_2 = X_1 \)
Properties of Genie: Smartness

• **Smart**: if when Gaussian inputs are used, genie does not improve sum capacity

\[ I(X_1^G; Y_1^G, S_1^G) = I(X_1^G; Y_1^G) \quad \text{and} \quad I(X_2^G; Y_2^G, S_2^G) = I(X_2^G; Y_2^G) \]

• Example: Genie that does not interact with receivers not useful!

• We want a genie that is both useful and smart

\[ C_{\text{sum}} = I(X_1^G; Y_1^G) + I(X_2^G; Y_2^G) \]
Quest for Divine Genie

- Restrict class of genies considered

\[ S_1 = X_1 + \eta W_1 \]
\[ S_2 = X_2 + \eta W_2 \]

- If \( W_i \) independent of \( Z_i \), \( i = 1, 2 \)
  - genie is useful in proving one bit result of Etkin, Tse, and Wang
  - Useful and asymptotically smart!
Geometric Representation of Genie

- \( \cos \theta \): Correlation coefficient between \( I + Z \) and \( \eta W \)
  (equivalently between \( Z \) and \( W \))
Inside green curve: useful
Condition: Useful Genie

- Mutual Information for the genie-aided channel

\[ I(X^n_i; Y^n_i, S^n_i) = I(X^n_i; S^n_i) + I(X^n_i; Y^n_i|S^n_i) \]
\[ \leq h(S^n_i) - n h(S_{iG}|X_{iG}) + n h(Y_{iG}|S_{iG}) - h(Y^n_i|X^n_i, S^n_i) \]

- Terms that are not maximized by Gaussian inputs:

\[ h(S^n_1) = h(hX^n_1 + h\eta W^n_1) \]
\[ h(Y^n_2|X^n_2, S^n_2) = h(hX^n_1 + Z^n_2|W^n_2) \]

and similar terms obtained by swapping “1” and “2”

- Difference between terms is maximized by \( X_{1G} \) (worst case noise result) if:

\[ |h\eta| \leq \sqrt{1 - \cos^2 \theta} \]
On the blue line: smart
Green curve intersects with blue line:
Treating interference as noise is optimal
Divine Genie

\[ h(1 + h^2P) \leq 0.5 \]

Intersection if \( h(1 + h^2P) \leq 0.5 \)

\[ \sqrt{1 + h^2P} \]

\[ \frac{0.5}{h(\sqrt{1+h^2P})} \]

Also [Shang, Kramer, Chen 08] & [Motahari, Khandani 08]
INR = $h^2P$ and SNR = $P$
Extensions – Outer Bound on Capacity Region

The diagram illustrates the capacity region of a communication system, focusing on the outer bounds. The axes are labeled as $R_1$ and $R_2$, with $R_1$ on the horizontal axis and $R_2$ on the vertical axis. The curves represent different bounds:

- **HK Inner Bound** (dashed black line)
- **ETW Outer Bound** (green line)
- **Broadcast Outer Bound** (blue dashed line)
- **New Outer Bound** (red line)

The graph provides a visual comparison of these bounds, highlighting the outer bound's performance in terms of capacity region.
Is treating interference as noise still optimal in low interference regime?

Yes, using same “scalar” genie we can find threshold below which treating interference as noise is optimal
Many-One and One-Many Channels

\[
\sum_{i=2}^{M} h_{1i}^2 \leq 1
\]

Treating interference as noise is optimal in low interference regime.
Symmetric $M$-User Interference Channel

- Interference threshold is characterized by
  \[ \hat{h}(1 + \hat{h}^2P) \leq 0.5, \text{ where } \hat{h}^2 = (M - 1)h^2 \]

- Threshold on total interference at receiver below which it is optimal to treat interference as noise remains constant as $M$ increases!
Case Study – Three User Symmetric Channel

- Requires new genie construction
- No explicit equation for threshold on interference parameter $h$
- But can compute admissible values of $h$ for given $P$ numerically
Interference Threshold

SNR vs INR Threshold for Two and Three Users

Two users
Three users
Extension to MIMO Interference Channels

Input covariance constraints: $Q_i \succeq 0, \operatorname{Tr}(Q_i) \leq P$
Genie-Aided MIMO Interference Channel

\[ S_i = H_c X_i + W_i \]

\[
\begin{bmatrix}
Z_i \\
W_i
\end{bmatrix}
\sim \mathcal{N}\left(0, \begin{bmatrix}
\Sigma & A \\
A^\top & \tilde{\Sigma}
\end{bmatrix}\right)
\]

Genie parameters: \( \Psi = \{\tilde{\Sigma}, A\} \)
Genie-aided outer bound

\[
\begin{align*}
\mathcal{C}_{\text{sum}} & \leq & \mathcal{C}_{\text{sum}}^{\text{GA-IC}}(\Psi) \\
\geq & & \text{Useful Genie} \equiv \text{Genie}
\end{align*}
\]

\[
\max_Q \sum_i I(X_{iG}; Y_{iG}) \quad \text{Smart Genie} \quad ? \quad \max_Q \sum_i I(X_{iG}; Y_{iG}, S_{iG})
\]
Useful and Smart Genie Conditions

Useful Genie: If $\tilde{\Sigma} \preceq \Sigma - A\tilde{\Sigma}^{-1}A^\top$ then:

$$C_{\text{sum}}^{\text{GA-IC}}(\Psi) = \max_Q R_{\text{TIN}}^{\text{GA-IC}}(Q, \Psi)$$

Smart Genie: If $(A^\top(H_cQH_c^\top + \Sigma)^{-1}H_d - H_c)Q = 0$, then

$$R_{\text{TIN}}^{\text{IC}}(Q) = R_{\text{TIN}}^{\text{GA-IC}}(Q, \Psi)$$

i.e., the genie is smart w.r.t. input covariance $Q$. 
Genie-aided outer bound: Fix $Q$

$C_{\text{sum}}(Q) \leq C_{\text{GA-IC sum}}(Q, \Psi)$

$R_{\text{TIN}}^{IC}(Q) \geq R_{\text{TIN}}^{GA-IC}(Q, \Psi)$

**Sufficient Condition:** Treating interference as noise is sum rate optimal if for every $Q$ satisfying power constraint, there exists a genie that is useful and smart w.r.t. $Q$ [Shang, Chen, Kramer & Poor, Allerton 08]
Low Interference Regime – Simpler Condition

**Theorem**

Let $Q^*$ be a global maximum of $R_{TIN}^{IC}(Q)$. If

- there exists a genie that is useful and smart w.r.t. $Q^*$
- $Q^*$ is full rank

then the channel is in low interference regime, i.e.,

$$C_{sum} = R_{TIN}^{IC}(Q^*).$$
Proof Illustration

\[ R_{GA-IC} \]

\[ R_{IC} \]

\[ Q^* \]
MISO Channel

$\theta$: angle between $\overrightarrow{d}$ and $\overrightarrow{c}$. 
Application of General MIMO Result?

- Is $Q^* = \arg \max R_{\text{TIN}}^{IC}(Q)$ full rank?
  
  No! $Q^*$ is unit rank

- Beamforming along direction $b$ is optimal, i.e.,

  $$Q^* = Pbb^T$$

  where $b$ is generalized eigenvector of matrix pair

$$\left( Pdd^T, I + h^2 Pcc^T \right)$$

corresponding to largest generalized eigenvalue $\lambda_{\text{max}}$
Genie-aided outer bound

$$C_{\text{sum}} \leq C_{\text{sum}}^{\text{GA-IC}}(\Psi)$$

Useful Genie

$$\geq \max_Q R_{\text{TIN}}^{\text{IC}}(Q)$$

$$= \max_Q R_{\text{TIN}}^{\text{GA-IC}}(Q, \Psi)$$

$$\leq \max_Q R_{\text{TIN}}^{\text{relaxed}}(Q, \Psi)$$

Additional Constraints

$$= R_{\text{TIN}}^{\text{IC}}(Q^*)$$

Smart Genie w.r.t. $Q^*$

$$= R_{\text{relaxed}}(Q^*, \Psi)$$
Outer Bound

![Graph showing the relationship between the angle between the Beamforming vector and the cross channel vector, and the sum rate. Graph includes lines for achievable rate, genie-aided outer bound, and relaxed outer bound.](image)
Low Interference Regime

**Theorem**

The sum capacity of the MISO interference channel is achieved by using Gaussian inputs, transmit beamforming, and treating interference as noise at the receivers, and is given by

\[
C_{\text{sum}}^{IC} = \frac{1}{2} \log \left( 1 + \frac{P \cos^2 \theta}{1 + h^2 P} + P \sin^2 \theta \right)
\]

if the channel gain parameter \( h \) satisfies:

\[
|h| < h_0(\theta, P)
\]

with \( h_0(\theta, P) \) being the positive solution to the implicit equation

\[
h^2 - \sin^2 \theta = \left( \frac{\cos \theta}{1 + h^2 P} - h \right)_+^2.
\]
Low Interference Regime

For $\theta = 0$, $h(1 + h^2P) \leq 0.5$ (SISO result)
Theorem

For any $P$, if

\[ h \leq \sin(\theta) \]

then sum capacity of MISO interference channel is achieved by using Gaussian inputs, transmit beamforming, and treating interference as noise at receivers.
**SIMO Channel**

$\theta$: angle between $d$ and $c$.

No covariance optimization
Improved usefulness condition

- \( h(h_c X + W_1) - h(h_c X + W_2) \) maximized by Gaussian distribution?

- \( \Sigma_1 \preceq \Sigma_2 \) is a sufficient condition, but not necessary.

- We can show that \( (c^\top \Sigma_1^{-1} c)^{-1} \leq (c^\top \Sigma_2^{-1} c)^{-1} \) is sufficient.
Genie Construction

- Symmetric genie signal

\[ S_i = h_cX_i + W_i \]

\[ \begin{bmatrix} \frac{Z_i}{W_i} \end{bmatrix} \sim \mathcal{N} \left( 0, \begin{bmatrix} I & A \end{bmatrix} \tilde{\Sigma} \right) \]

- Useful Genie:

\[ \left( c^T \tilde{\Sigma}^{-1} c \right)^{-1} \leq \left( c^T \left( I - A\tilde{\Sigma}^{-1} A^T \right)^{-1} c \right)^{-1} \]

- Smart Genie:

\[ A^T (h^2 P_{cc}^T + I)^{-1} d - h_c = 0 \]

- Goal: Find \( A \) and \( \tilde{\Sigma} \) that result in largest threshold on \( h \) for low interference regime
Low Interference Regime

Theorem

The sum capacity of the SIMO interference channel is achieved by using Gaussian inputs, receive beamforming, and treating interference as noise at the receivers, and is given by

\[ C_{\text{sum}}^{IC} = \frac{1}{2} \log \left( 1 + \frac{P \cos^2 \theta}{1 + h^2 P} + P \sin^2 \theta \right) \]

if the channel gain parameter \( h \) satisfies:

\[ |h| < h_0(\theta, P) \quad (3) \]

with \( h_0(\theta, P) \) being the positive solution to the implicit equation

\[ h^2 - \sin^2 \theta = \left( \frac{\cos \theta}{1 + h^2 P} - h \right)_+^2. \quad (4) \]

Same threshold as the MISO channel
Low Interference Regime

For $\theta = 0$, $h(1 + h^2 P) \leq 0.5$ (SISO result)
Low Interference Regime - Simplified

Theorem

For any $P$, if

$$h \leq \sin(\theta)$$

then sum capacity of MISO interference channel is achieved by using Gaussian inputs, transmit beamforming, and treating interference as noise at receivers.
Interference Management – Lessons from IT

- Treating interference as noise is optimal as long as it is very weak (low) – counterpart of very strong interference regime.

- Low interference regime can be significant with multiple antennas.

- What if interference is not low (but still weak)?
  - SISO: Han-Kobayashi scheme with joint decoding in two-user case (one bit optimal [Etkin, Tse, Wang 07])
  - Simple schemes that exploit structure in interference without requiring joint decoding?
References


Part 3: Degrees of Freedom Characterization of Interference Channels
Information Theory for IC: State-of-the-art

- **Exact characterization**
  - Very hard problem, still open even after > 30 years

- **Approximate characterization**
  - Within constant number of bits/sec
  - Provides some architectural insights

- **Degrees of freedom (or multiplexing gain)**

  \[
  \text{DoF} = \lim_{{\text{SNR} \to \infty}} \frac{\text{sum capacity}}{\log \text{SNR}}
  \]

  - Pre-log factor of sum-capacity in high SNR regime
  - Number of interference free sessions per channel use
  - Simplest of the three, but can provide useful insight
$K$-user (SISO) Interference Channel

How many Degrees of Freedom (DoF)?
Degrees of Freedom with Orthogonalization

- One active user per channel use
  - Every user gets an interference free channel once every $K$ channel uses
  - DoF per user is $1/K$; total DoF equals 1

- Special Case: $K = 2$
  - Can easily show that outer bound on DoF equals 1

$\implies$ TDMA optimal from DoF viewpoint for $K = 2$
Degrees of Freedom for general $K$

- Outer Bound on DoF \cite{Host-Madsen, Nosratinia '05}
  - There are $K(K - 1)/2$ pairs and each user appears in $(K - 1)$ pairs
  - Thus DoF $\leq K/2$ or per user DoF $\leq 1/2$

- Amazingly, this outer bound is achievable via linear interference suppression!

Interference Alignment \cite{Cadambe & Jafar '08}
Interference Channel with Tx/Rx Linear Coding

\[ \begin{bmatrix} U_1 & 0 & 0 \\ 0 & U_2 & 0 \\ 0 & 0 & U_3 \end{bmatrix}^\dagger \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix} \begin{bmatrix} V_1 & 0 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & V_3 \end{bmatrix} \]

Receive Beams  Channel  Transmit Beams

End-to-End matrix is Diagonal  \(\implies\) No Interference!

\# streams = Size of the Diagonal matrix
DoF of Linear Strategies

\[
\begin{bmatrix}
U_1 & 0 & 0 \\
0 & U_2 & 0 \\
0 & 0 & U_3
\end{bmatrix}^\dagger
\begin{bmatrix}
H_{1,1} & H_{1,2} & H_{1,3} \\
H_{2,1} & H_{2,2} & H_{2,3} \\
H_{3,1} & H_{3,2} & H_{3,3}
\end{bmatrix}
\begin{bmatrix}
V_1 & 0 & 0 \\
0 & V_2 & 0 \\
0 & 0 & V_3
\end{bmatrix}
\]

\[H_{i,j} : NT \times NT \text{ block-diagonal matrix}\]

- (Symmetric) MIMO:
  \[N = \# \text{ antennas}\]
- Symbol Extensions (Time or Frequency)
  \[T = \# \text{ symbol extensions}\]

\[\text{DoF} (T) = (\#\text{streams})/T\]
Interference Alignment with Symbol Extensions

\[ H_{i,j} = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \]

3 Symbol Extensions

4 interference free streams \( \implies \) PUDoF = 4/9
Interference Alignment with Symbol Extensions

- Set $\mathbf{v}_{1a}$ to some fixed direction
- Align $\mathbf{v}_2$ with $\mathbf{v}_{11}$ at Rx 3:
  \[
  \mathbf{H}_{3,2} \mathbf{v}_2 = \mathbf{H}_{3,1} \mathbf{v}_{1a}
  \]
  \[
  \mathbf{v}_2 = \mathbf{H}_{3,2}^{-1} \mathbf{H}_{3,1} \mathbf{v}_{1a}
  \]
- Align $\mathbf{v}_3$ with $\mathbf{v}_2$ at Rx 1:
  \[
  \mathbf{H}_{1,3} \mathbf{v}_3 = \mathbf{H}_{1,2} \mathbf{v}_2
  \]
  \[
  \mathbf{v}_3 = \mathbf{H}_{1,3}^{-1} \mathbf{H}_{1,2} \mathbf{v}_2
  \]
- Align $\mathbf{v}_{1b}$ with $\mathbf{v}_3$ at Rx 2:
  \[
  \mathbf{H}_{2,1} \mathbf{v}_{1b} = \mathbf{H}_{2,3} \mathbf{v}_3
  \]
  \[
  \mathbf{v}_{1b} = \mathbf{H}_{2,1}^{-1} \mathbf{H}_{2,3} \mathbf{v}_3
  \]
Interference Alignment with Symbol Extensions

- Signaling matrix for transmitter $\ell$: $V_\ell = [v_{\ell 1} \ v_{\ell 2} \cdots v_{\ell m}]$
- Fix $\mathcal{I}_k$ as the interference space at receiver $k$
- Alignment conditions are:
  - Receiver 1
    \[
    H_{1,1} V_1 \cap \mathcal{I}_1 = \emptyset \quad H_{2,1} V_1 \subseteq \mathcal{I}_2 \quad H_{1,2} V_2 \subseteq \mathcal{I}_1 \quad H_{2,2} V_2 \cap \mathcal{I}_2 = \emptyset \\
    \vdots \quad \vdots \quad \vdots \quad \vdots \\
    H_{1,K} V_K \subseteq \mathcal{I}_1 \quad H_{2,K} V_K \subseteq \mathcal{I}_2 \quad H_{K,K} V_K \cap \mathcal{I}_K = \emptyset
    \]
  - Receiver 2
  - Receiver $K$
- For alignment we require $m/2$ dimensions each for $V_k$ and $\mathcal{I}_k$
- May not be possible in general
Asymptotic Interference Alignment

- Enumerate all \( S \triangleq K(K - 1) \) cross-channels with single index:

\[
\mathcal{T} = \{T_i\} = \{H_{k,\ell} : k \neq \ell\}
\]

- Use same signal space \( \mathcal{V}^{(m)} \) at all Tx’s, defined recursively as:

\[
\begin{align*}
\mathcal{V}^{(0)} &= \{1\} \\
\mathcal{V}^{(m)} &= \left\{ v, T_1 v, \ldots, T_S v : v \in \mathcal{V}^{(m-1)} \right\} \\
&= \left\{ T_{1}^{\alpha_1} T_{2}^{\alpha_2} \cdots T_{S}^{\alpha_S} 1 : \alpha_1 + \alpha_2 + \cdots + \alpha_S \leq m \right\}
\end{align*}
\]

- Dimension of signal space (\# symbol extensions):

\[
\left| \mathcal{V}^{(m)} \right| = \binom{m + S}{m}
\]
Asymptotic Interference Alignment

- **Interference space** at Rx $k$ is $\mathcal{I}_k = \bigcup_{\ell \neq k} H_{k,\ell} \mathcal{V}(m) \subset \mathcal{V}(m+1)$

- **Desired signal space** at Rx $k$ is $H_{k,k} \mathcal{V}(m)$

- Almost surely, no overlap between desired signal space and interference space as long as channel coefficients are changing over symbol extensions and are generic

- Also

\[
\frac{|\mathcal{V}(m)|}{|\mathcal{V}(m+1)|} = \frac{(m+S)}{m+1} = \frac{m + 1}{m + 1 + S} \quad m \to \infty \quad \frac{1}{1}
\]

- Desired signal space gets half the dimensions asymptotically:

\[
\frac{|H_{k,k} \mathcal{V}(m)|}{|H_{k,k} \mathcal{V}(m)| + |\mathcal{I}_k|} \quad m \to \infty \quad \frac{1}{2}
\]
PUDoF of 0.5 is achieved asymptotically
Complexity of asymptotic Interference Alignment

[Choi, Jafar, and Chung, ’09]
Interference Alignment Summary

+ Achieves optimal PUDoF for fully connected channel

- Requires global channel state information (CSI)

- Requires large number of symbol extensions
References


Part 4: Finite Diversity and Iterative Algorithms
Finite Diversity and Interference alignment

• Asymptotic interference alignment schemes provide degree of freedom gains by letting number of symbol extensions go to infinity

• In practice, we only have finite number of symbol extensions from diversity in time, frequency, or multiple antennas

• How do we apply idea of interference alignment to finite diversity case?
Outline

• Look at two cases of IA with finite diversity
  1 Spatial diversity through multiple antennas
  2 Time and frequency diversity

• Provide theoretical guarantees for DoF using IA schemes

• Iterative algorithms inspired by IA for MIMO IC
Constant MIMO Channel

- Fully connected $K$-user MIMO interference channel
- $N_{t}^{(k)}$ transmit antennas and $N_{r}^{(k)}$ receive antennas for user $k$
- Usual channel model for receiver $k$ given by

$$Y_{k} = \sum_{j=1}^{K} H_{kj} X_{j} + Z_{k}$$

- $Y_{k}$ - signal at receiver $k$ ($N_{r}^{(k)} \times 1$)
- $H_{kj}$ - channel from transmitter $j$ to receiver $k$ ($N_{r}^{(k)} \times N_{t}^{(j)}$)
- $X_{j}$ - signal sent by transmitter $j$ ($N_{t}^{(j)} \times 1$)
- $Z_{k}$ - noise at receiver $k$ ($N_{r}^{(k)} \times 1$)

- Channels $\{H_{kj}\}$ drawn once from continuous joint distribution hence name constant MIMO channel
Constant MIMO Channel
Interference Alignment for Constant MIMO Channel

- Consider linear transmit and receive strategies:

\[ U_k^\dagger Y_k = U_k^\dagger \sum_{j=1}^{K} H_{kj} V_j \tilde{x}_j + U_k^\dagger Z_k \]

- Transmitter constructs channel input \( X_j = V_j \tilde{x}_j \) with \( V_j \) size \( N_t^{(j)} \times d_j \)

- Receiver processes channel output \( U_k^\dagger Y_k \) with \( U_k \) size \( N_r^{(k)} \times d_k \)

- IA for Constant MIMO IC: Construct \( \{ V_k \} \) and \( \{ U_k \} \) such that receivers can zero force interference yielding interference free channels between each Tx/Rx pair \( k \)
Interference Alignment for Constant MIMO Channel

- Goal: to create $d_k$ interference free streams between Tx/Rx pair $k$
- To zero force interference, we need

$$ U_k^\dagger H_{kj} V_j = 0 \quad \forall \ k \neq j $$

- Zero forcing all interference at Rx $k$ yields effective channel

$$ U_k^\dagger Y_k = U_k^\dagger H_{kk} V_k \tilde{x}_k + U_k^\dagger Z_k $$

- This is MIMO channel with channel matrix $U_k^\dagger H_{kk} V_k$
- To create $d_k$ interference free channels, we need

$$ \text{rank} \left( U_k^\dagger H_{kk} V_k \right) = d_k $$
Interference Alignment for Constant MIMO Channel

• For IA in constant MIMO channel with $d_k$ interference free channels between Tx/Rx pair $k$, we need

$$\text{rank} \left( U_k^\dagger H_{kk} V_k \right) = d_k \quad \forall k$$

$$U_k^\dagger H_{kj} V_j = 0 \quad \forall k \neq j$$

• Choose transmit and receive vectors $\{V_k\}$ and $\{U_k\}$ using only interfering channels $\{H_{kj}\}_{k \neq j}$

• Combined with our assumption that $\{H_{kj}\}$ are drawn from joint distribution, (almost surely) it holds that

$$\text{rank} \left( U_k^\dagger H_{kk} V_k \right) = d_k$$

Rank condition is automatic
Solving IA Equations

• We only need to solve IA equations

\[ U_k^\dagger H_{kj} V_j = 0 \quad \forall k \neq j \]

When is this possible?

• Feasibility of solving IA equations will depend on \( K \), \( \{N_t^{(k)}\} \), \( \{N_r^{(k)}\} \), and \( \{d_k\} \)

• Easy to derive necessary conditions for solving IA equations using variable and equation counting [Yetis10, Razaviyayn12, Gonzalez12]

• For some specific system configurations, possible to derive sufficient conditions using algebraic geometry (branch of math focused on answering when polynomial equations have solutions)
Symmetric Constant MIMO IC - Necessary

• To illustrate necessary condition, consider symmetric constant MIMO IC:
  1. \( N_t^{(k)} = N_r^{(k)} = N \)
  2. \( d_k = d \)

• In order to solve IA equations, number of variables should be no less than number of equations
  • This intuitive idea can be proved rigorously using algebraic geometry or algebraic field theory

• The no interference equation for Tx j at Rx k

\[ U_k^\dagger H_{k,j} V_j = 0 \]

consists of \( d^2 \) equations

• There are total of \( K(K - 1)d^2 \) equations to ensure no interference
Symmetric Constant MIMO IC - Necessary

- Counting number of variables is little bit trickier
- Satisfying IA equations only depends on subspaces \( \{\text{span}(U_k)\} \) and \( \{\text{span}(V_k)\} \) and not particular choice of \( \{U_k\} \) and \( \{V_k\} \)
- To illustrate this point, suppose that \( U_k^\dagger H_{kj} V_j = 0 \quad \forall k \neq j \)
- For any other basis \( \{\tilde{V}_j\} \) and \( \{\tilde{U}_j\} \) of \( \{\text{span}(U_k)\} \) and \( \{\text{span}(V_k)\} \), there exist \( d \times d \) full rank matrices \( \{Q_k\} \) and \( \{P_k\} \) such that

\[
\begin{align*}
\tilde{V}_j &= V_j Q_j \\
\tilde{U}_k &= U_k P_k
\end{align*}
\]

- Then we have \( \tilde{U}_k^\dagger H_{kj} \tilde{V}_j = P_k^\dagger U_k^\dagger H_{kj} V_j Q_j = 0 \)
Symmetric Constant MIMO IC - Necessary

- Since satisfying IA equations only depends on subspaces spanned by \( \{V_k\} \) and \( \{U_k\} \), number of variables is less than \( 2K \times Nd \)
- By preceding argument, due to freedom in choosing \( P_k \) and \( Q_k \) in fact we have

\[
\dim(\text{span}(V_k)) = \dim(\text{span}(U_k)) = Nd - d^2 = (N - d)d
\]

- This implies that number of variables is \( 2K \times d(N - d) \)
- Necessary condition for IA in symmetric constant MIMO IC becomes

\[
(K + 1)d \leq 2N
\]

- This implies that

\[
\text{DoF} \leq K \left[ \frac{2N}{K + 1} \right] \leq 2N \frac{K}{K + 1} \leq 2N
\]
Solving IA Equations - Feasability

- This necessary condition is not always sufficient [Yetis10]
  - \( N_t^{(k)} = N_r^{(k)} = 3, \ d_k = 2, \) and \( K = 2 \) system satisfies necessary condition but is not achievable
- For symmetric constant MIMO channel with \( K \geq 3, \) necessary condition \((K + 1)d \leq 2N\) is sufficient [Bresler14]
- Necessary and sufficient conditions for feasibility of IA for other configurations too using similar ideas [Razaviyayn12,Ruan13,Gonzalez12]
- Conditions for feasibility of solving IA equations for constant MIMO IC not known in general
Finite Time and Frequency Diversity

- To model finite diversity, suppose that we have $L$ symbol extensions, equivalently a diagonal channel matrix

$$H_{kj} = \begin{bmatrix} H_{kj}(1) & 0 \\ & \ddots \\ & 0 & H_{kj}(L) \end{bmatrix}$$

- For IA in $K$-user IC, $L \geq 2^{K^2}$ sufficient [ÖzgürTse09]

- Three user case has been exactly characterized exactly using vector IA achievable scheme [BreslerTse09]

$$\text{DoF} = \frac{3}{2} \left( 1 - \frac{1}{4L - 2\lfloor L/2 \rfloor - 1} \right) = \mathcal{O} \left( \frac{3}{2} \left( 1 - \frac{1}{L} \right) \right)$$

- As $L \to \infty$, recover usual $\frac{3}{2}$ DoF
Finite Diversity and IA

• For $K \geq 4$ case with finite diversity, there are sum DoF upper bounds [LiÖzgür14]

$$\text{DoF} \leq \frac{K}{2} \left( 1 - \frac{1}{11\sqrt{L}} \right)$$

• Achievable scheme yields

$$\text{DoF} \geq \frac{K}{2} \left( 1 - \frac{C_1}{C_2 \sqrt{L/2}} \right)$$

with $C_1$ a constant and $C_2 = (K - 1)(K - 2) - 1$

• Substantial gap between upper bound and achievable scheme

• As $L \to \infty$, recover usual $\frac{K}{2}$ DoF
Time/Frequency vs. Spatial Diversity

- For time and frequency diversity, when amount of diversity ($L$) is large, $\text{DoF} \approx \frac{K}{2}$
- DoF gains for time/frequency diversity scale with $K$ but for $K \geq 4$ may need large $L$ to get close to $\frac{K}{2}$
  - Coherence time of channel can be an issue for large $L$
- For spatial diversity in symmetric constant MIMO IC, $\text{DoF} \approx 2N$
- DoF gains for spatial diversity do not scale with $K$ but no need to code over a large number of time/frequency slots
- Finally, for time-varying symmetric MIMO channel with $N$ antennas at all Tx/Rx [CadambeJafar08]

\[
\text{DoF} = \frac{KN}{2}
\]
IA Inspired Algorithms for Constant MIMO IC

- Examined when we can solve IA equations

\[
\text{rank} \left( U_k^\dagger H_{kk} V_k \right) = d_k \quad \forall k
\]

\[
U_k^\dagger H_{kj} V_j = 0 \quad \forall k \neq j
\]

- Analysis does not tell us how to actually solve IA equations
- Iterative algorithms based on idea of IA for constant MIMO IC
- Channel State Information: Assume each transmitter and receiver knows all connected channels
Look at two algorithms inspired by IA [Gomadam08]

Designing transmit vectors to minimize interference is difficult because each transmit vector affects interference at all receivers

- Similarly, designing transmit vectors to maximize sum rate is non-convex problem and difficult to solve

In contrast, each receive vector is affected only by interference seen at receiver and is easy to design

Idea of both algorithms is to only design receive vectors but alternate the direction of communication

- Role of transmit and receive vectors alternates
- Appropriate for TDD systems with natural reversal of directions of communication
Constant MIMO IC Algorithm Overview

Forward Direction - Design Receive Vectors Reverse Direction of Communication:

\[ \hat{V}_k(n) = \hat{U}_k(n) \]

Reverse Direction - Design Receive Vectors Reverse Direction of Communication and Repeat:
Min Leakage Algorithm

- For fixed transmit vectors \( \{V_k\} \), interference leakage power at receiver \( k \) given by \( I_k = \text{trace}(U_k^\dagger Q_k U_k) \) with

\[
Q_k \triangleq P \sum_{j=1}^{K} H_{kj} V_j V_j^\dagger H_{kj}^\dagger
\]

- If \( I_k = 0 \), then we achieve IA
- To minimize interference leakage power, choose \( d_k \) least dominant eigenvectors of \( Q_k \), i.e., those eigenvectors corresponding to \( d_k \) smallest eigenvalues of \( Q_k \)
- Alternate directions reversing role of transmit and receive vectors
- Sum interference leakage power converges
Min Leakage Algorithm

1. Start with arbitrary $\{\vec{V}_k(0)\}$
2. Set $\vec{U}_k(n)$ to be $d_k$ least dominant eigenvectors of $\vec{Q}_k(n)$
3. Reverse direction of communication
   \[ \vec{V}_k(n) = \vec{U}_k(n) \]
4. Set $\vec{U}_k(n)$ to be $d_k$ least dominant eigenvectors of $\vec{Q}_k(n)$
5. Reverse direction of communication and repeat
   \[ \vec{V}_k(n + 1) = \vec{U}_k(n) \]
Max SINR Algorithm

- At high SNR, Min Leakage provides good performance - interference limited regime
- At lower SNR, Min Leakage does not provide good performance - noise becomes more important
- Max SINR same form as Min Leakage algorithm but with MMSE receive vectors

\[ U_k = \left( I + P \sum_{\substack{j=1 \ j \neq k}}^{K} H_{kj} V_j V_j^\dagger H_{kj}^\dagger \right)^{-1} H_{kk} V_k \]
Max SINR Algorithm

- At high SNR, MMSE filter becomes zero forcing filter - focuses on interference
- At low SNR, MMSE filter becomes matched filter - focuses on noise
- At intermediate SNR, MMSE filter trades off between interference and noise
- Max SINR has better performance at low SNR than Min Leakage or other purely IA oriented algorithms
- No proof of convergence but appears to converge in practice
Performance Comparison

Similar performance at high SNR Max SINR much better at low SNR
Other Algorithms

- Min Leakage and Max SINR make sense in TDD system due to natural alternation of transmitting and receiving
- IA Algorithms for FDD channels have been developed [PetersHeath09]
  - Similar in spirit to Min Leakage algorithm but allows for FDD
- Convergent version of Max SINR [WilsonVVV11]
  - Power control step to produce non-decreasing sum rate
  - Same performance as Max SINR in simulations
- Several other similar iterative algorithms inspired by IA in literature
References


References


9 C.T. Li, and A. Ozgur, “Channel Diversity needed for Vector Interference Alignment”, on arXiV, 2014


Part 5: Coordinated Multi-Point Transmission
Channel State Information known at all nodes.
Coordinated Multi-Point (CoMP) Transmission

Messages are jointly transmitted using multiple transmitters.

\[ W_1 \xrightarrow{Tx_1} Rx_1 \xrightarrow{\hat{W}_1} \]
\[ W_2 \xrightarrow{Tx_2} Rx_2 \xrightarrow{\hat{W}_2} \]
\[ W_3 \xrightarrow{Tx_3} Rx_3 \xrightarrow{\hat{W}_3} \]
CoMP Transmission

- Each message is jointly transmitted using $M$ transmitters
- Message $i$ is transmitted jointly using the transmitters in $T_i$
- For all $i \in [K]$, $|T_i| \leq M$
- We consider all message assignments that satisfy the cooperation constraint
Degrees of Freedom (DoF)

\[
\text{DoF} = \lim_{\text{SNR} \to \infty} \frac{\text{sum capacity}}{\log \text{SNR}}
\]

Objective: Determine the DoF as a function of \(K\) and \(M\)

\[
PUDoF(M) = \lim_{K \to \infty} \frac{\text{DoF}(K, M)}{K}
\]

Is \(PUDoF(M) > PUDoF(1)\) for \(M > 1\)?
Example: Two-user Interference Channel

No Cooperation, DoF=1, Time Sharing

Full Cooperation, DoF=2, ZF Transmit Beamforming
No Cooperation \((M = 1)\)

- For \(M = 1\), outer bound = \(K/2\)
- The outer bound can be achieved by jointly coding across multiple parallel channels [Cadambe & Jafar '08]:

\[
\text{DoF}(K, M = 1) = \lim_{L \to \infty} \frac{\text{DoF}(K, M = 1, L)}{L} = \frac{K}{2}
\]

where \(L\) is the number of parallel channels

**Corollary**

*Without cooperation, the Per User DoF number is given by*

\[
PUDoF(M = 1) = \frac{1}{2}
\]
Full Cooperation \((M = K)\)

- In this case, the channel is a MISO Broadcast channel.
- Each message is available at \(K\) antennas, and hence, can be canceled at \(K - 1\) receivers.
- Each user achieves 1 DoF,

\[
\text{DoF}(K, M = K) = K.
\]

What happens with partial cooperation \((1 < M < K)\)?
Clustering

No Degrees of Freedom Gain
Spiral Message Assignment

\[ \mathcal{T}_i = \{i, i + 1, \ldots, i + M - 1\} \]
Spiral Assignment: Matrix Interpretation

\[
\begin{bmatrix}
U^{[1]} & 0 & 0 \\
0 & U^{[2]} & 0 \\
0 & 0 & U^{[3]}
\end{bmatrix}^H 
\begin{bmatrix}
H^{[1,1]} & H^{[1,2]} & H^{[1,3]} \\
H^{[2,1]} & H^{[2,2]} & H^{[2,3]} \\
H^{[3,1]} & H^{[3,2]} & H^{[3,3]}
\end{bmatrix} 
\begin{bmatrix}
V^{[1]} & 0 & 0 \\
0 & V^{[2]} & 0 \\
0 & 0 & V^{[3]}
\end{bmatrix}
\]

Receive Beams  Channel  Transmit Beams
Spiral Assignment: Matrix Interpretation

\[
egin{bmatrix}
U^{[1]} & 0 & 0 \\
0 & U^{[2]} & 0 \\
0 & 0 & U^{[3]}
\end{bmatrix}^H
\begin{bmatrix}
H^{[1,1]} & H^{[1,2]} & H^{[1,3]} \\
H^{[2,1]} & H^{[2,2]} & H^{[2,3]} \\
H^{[3,1]} & H^{[3,2]} & H^{[3,3]}
\end{bmatrix}
\begin{bmatrix}
V^{[1]} & 0 & V^{[3]} \\
V^{[1]} & V^{[2]} & 0 \\
0 & V^{[2]} & V^{[3]}
\end{bmatrix}
\]

Receive Beams \quad Channel \quad Transmit Beams

\( M \): \# of non-zero blocks in the columns of \( V \)
Example: \( K = 3, M = 2 \)

\[
PUDoF = \frac{2}{3}
\]
Spiral Message Assignment: Results

Theorem

The DoF of interference channel with a spiral message assignment satisfies

\[ \frac{K + M - 1}{2} \leq \text{DoF}(K, M) \leq \left\lceil \frac{K + M - 1}{2} \right\rceil \]

Proof of Achievability: First \( M - 1 \) users are interference-free, and interference occupies half the signal space at each other receiver.

Generalizes the Asymptotic Interference Alignment scheme.
Vector Space Interference Alignment [Cadambe-Jafar ’08]

- Asymptotic interference alignment works only if the \((\text{Signal} + \text{Interference})\) matrix is full rank

- The generic channel coefficients assumption (they have a joint pdf) is crucial

- With CoMP, the same point to point channel can carry both desired and interfering signals

- It is not clear whether the generic channel coefficients assumption suffices to prove full rankness of the \((\text{Signal} + \text{Interference})\) matrix
Key Tool: Algebraic Independence

Consider the system of polynomial equations

\[ s_1 = f_1(t_1, t_2, \cdots, t_n) \]
\[ s_2 = f_2(t_1, t_2, \cdots, t_n) \]
\[ \vdots \]
\[ s_m = f_m(t_1, t_2, \cdots, t_n). \]

**Definition**

The polynomials \( f_1, f_2, \cdots, f_m \) are said to be algebraically dependent if and only if there exists an annihilating polynomial \( F(s_1, s_2, \cdots, s_m) \) such that \( F(f_1, f_2, \cdots, f_m) = 0. \)
The polynomials $f_1, f_2, \cdots, f_m$ are algebraically independent if and only if the Jacobian matrix

$$
\left( \frac{\partial f_i}{\partial t_j} \right)_{1 \leq i \leq m, 1 \leq j \leq n}
$$

has full structural row rank equal to $m$. 

Jacobian Criterion
Consider the system of polynomial equations

\[ s_1 = f_1(t_1, t_2, \cdots, t_n) \]
\[ s_2 = f_2(t_1, t_2, \cdots, t_n) \]
\[ \vdots \]
\[ s_m = f_m(t_1, t_2, \cdots, t_n). \]

**Theorem**

*If the variables \( \{t_1, \ldots, t_n\} \) have a continuous joint pdf and the polynomials \( f_1, f_2, \ldots, f_m \) are algebraically independent, then the variables \( \{s_1, \ldots, s_m\} \) have a continuous joint pdf.*
Outline of the Achievable Scheme

Approach:

1. ZF Step: Exploit cooperation to transform the interference channel into a *derived* channel (with single-point transmission)

2. IA Step: Use the known IA techniques to design beams for derived channel

3. Prove that the asymptotic IA step works for generic channel coefficients
Example: $M = 2$
Derived Channel Example: \( M = 2 \)

Asymptotic Interference Alignment is used to pack the interference in half the signal space at Rx 2 and 3.
Asymptotic IA for Derived Channel

• (Signal + Interference) matrix has full column rank if the variables \( \{g^{(m)}_{i,j}\} \) have a continuous joint pdf

• \( \{g^{(m)}_{i,j}\} \) have a continuous joint pdf because \( \{h_{i,j}\} \) have a continuous joint pdf and the polynomials defining the transformations from original to derived channel coefficients are algebraically independent

Prove algebraic independence using Jacobian Criterion
DoF Outer Bound

\[ W_1 \xrightarrow{Tx1} R\hat{x}1 \xrightarrow{\hat{W}_1} \]
\[ W_2 \xrightarrow{Tx2} R\hat{x}2 \xrightarrow{\hat{W}_2} \]
\[ W_3 \xrightarrow{Tx3} R\hat{x}3 \xrightarrow{\hat{W}_3} \]
\[ W_4 \xrightarrow{Tx4} R\hat{x}4 \xrightarrow{\hat{W}_4} \]
\[ W_5 \xrightarrow{Tx5} R\hat{x}5 \xrightarrow{\hat{W}_5} \]
DoF Outer Bound

Generic channel coefficients $\rightarrow$ DoF $\leq 3$
DoF Outer Bound: Bipartite Graph Representation

\[ |\text{Neighborhood} \{ \text{Tx1}, \text{Tx2} \} | = 3 \geq K - |\{ \text{Tx1}, \text{Tx2} \} | \implies \text{DoF} \leq 3 \]
DoF Outer Bound: Bipartite Graph Representation

Theorem

\[
\text{DoF} \leq \min_{S \subseteq \{1, \ldots, K\}} \max (|\text{Neighborhood}(S)|, K - |S|)
\]
DoF Outer Bound: Bipartite Graph Representation

Definition

\[
\text{DoF}_{out}(K, M) = \max_{\mathcal{T}_i} \min_{S \subseteq \{1, \ldots, K\}} \max(|\text{Neighborhood}(S)|, K - |S|)
\]

\[
\text{PUDoF}_{out}(M) = \lim_{K \to \infty} \frac{\text{DoF}_{out}(K, M)}{K}
\]

\[
\text{DoF}(K, M) \leq \text{DoF}_{out}(K, M)
\]

\[
\text{PUDoF}(M) \leq \text{PUDoF}_{out}(M)
\]
DoF Outer Bound: Results

Definition

We say that a message assignment satisfies a local cooperation constraint if and only if \( \exists r(K) = o(K) \), and for all \( K \)-user channels,

\[
\mathcal{T}_i \subseteq \{ i - r(K), i - r(K) + 1, \ldots, i + r(K) \}, \forall i \in [K]
\]

Theorem

*With the restriction to local cooperation,*

\[
\text{PUDoF}_{loc}(M) = \frac{1}{2}
\]

Local cooperation cannot achieve a scalable Dof gain
DoF Outer Bound: Results

**Theorem**

For $M \geq 2$,

\[ \text{PUDoF}(M) \leq \text{PUDoF}_{out}(M) \leq \frac{M - 1}{M} \]

**Corollary**

\[ \text{PUDoF}(2) = \frac{1}{2} \]

Assigning each message to two transmitters **cannot** achieve a scalable DoF gain
Fully Connected IC with CoMP: Summary

- Considered general cooperation constraint that limits the number of transmitters at which each message can be available.

- Asymptotic Interference Alignment can be used to achieve DoF gains.

- Dual results can be obtained with receiver cooperation but requires sharing of analog received signals.

- Iterative Max SINR type algorithms can be designed in constant MIMO channel setting [WilsonVVV-14].

- Scalable DoF gains cannot be achieved through:
  - Local Cooperation
  - Arbitrary assignment of each message to two transmitters.
References


Part 6: Locally Connected Networks
Locally Connected Model

Tx $i$ is connected to receivers $\{i, i+1, \ldots, i+L\}$.

$L = 2$

Wyner Model: $L = 1$
Results for Wyner Model [Lapidoth-Shamai-Wigger '07]

Backhaul load factor $= 1$

PUDoF $(L=1, M=2) = 2/3 > 1/2$
Results for Wyner Model  [Lapidoth-Shamai-Wigger '07]

- Spiral transmit sets: $\mathcal{T}_i = \{i, i + 1, \ldots, i + M - 1\}$

- $\text{PUDoF}(L = 1, M) = \frac{M}{M+1}$

  Backhaul load factor $= \frac{M}{2}$

- Local cooperation can achieve PUDoF gains for locally connected channels

- Achievable scheme relies on only:
  - Zero-forcing transmit beamforming
  - Local CSI
  - Fractional reuse

Is spiral message assignment optimal?
Example: No Cooperation

\[ \text{PUDoF}(L = 1, M = 1) = \frac{1}{2} \]

\[ \text{PUDoF}(L = 1, M = 1) = \frac{2}{3} \]

Interference-aware message assignment + Fractional reuse
Locally Connected IC with CoMP: Main Result

Theorem

Under the general cooperation constraint \(|T_i| \leq M, \forall i \in \{1, 2, \ldots, K\}\),

\[
\frac{2M}{2M + L} \leq \text{PUDoF}(L, M) \leq \frac{2M + L - 1}{2M + L}
\]

and the optimal message assignment satisfies a local cooperation constraint.

Corollary

\[
\text{PUDoF}(L = 1, M) = \frac{2M}{2M + 1}
\]
DoF Achieving Scheme

Backhaul load factor = 6/5

PUDoF \((L=1, M=2) = 4/5 > 2/3\)
DoF Outer Bound

Have to consider all possible message assignments satisfying

\[ |T_i| \leq M, \forall i \in [K] \]

1. First simplify the combinatorial aspect of the problem by identifying *useful message assignments*

2. Then derive an equivalent model with fewer receivers and same DoF
An assignment of a message $W_x$ to a transmitter $T_y$ is useful only if one of the following conditions holds:

1. **Signal delivery:** $T_y$ is connected to the designated receiver $R_x$

2. **Interference mitigation:** $T_y$ is interfering with another transmitter $T_z$, both carrying the message $W_x$
Assigning $W_3$ to Tx1 is not useful.
Corollary

An assignment of a message $W_x$ to a transmitter $T_y$ is useful only if there exists a chain of interfering transmitters carrying $W_x$ that includes $T_y$ and another transmitter $T_z$ that is connected to $R_x$.

Proves optimality of local cooperation
Extensions: Multiple-Antenna Transmitters

Theorem

For the locally connected interference channel with $N$-antenna transmitters, if $MN \geq L + M$ then $PUDoF(L, M, N) = 1$, otherwise,

$$\frac{2MN}{M(N + 1) + L} \leq PUDoF(L, M, N) \leq \frac{M(N + 1) + L - 1}{M(N + 1) + L}$$

and the optimal message assignment satisfies a local cooperation constraint.
CoMP Transmission for IC: Summary

- Local Cooperation
  - no PUDoF gain for fully connected channel
  - is optimal for locally connected channel

- Interference aware message assignments allow for higher throughput

- Fractional reuse and zero-forcing transmit beam-forming are sufficient to achieve PUDoF gains, without need for symbol extensions and interference alignment


Part 7: Cellular Networks and Backhaul Load Constraint
Backhaul Load Constraint

More natural cooperation constraint that takes into account overall backhaul load:

\[ \sum_{i \in [K]} |\mathcal{T}_i| \leq B \]

Solution under transmit set size constraint \(|\mathcal{T}_i| \leq M, \forall i \in [M]|\), can be used to provide solutions under backhaul load constraint
Wyner’s Model with Backhaul Load Constraint

Theorem

Under cooperation constraint
\[ \sum_{i \in [K]} |T_i| \leq BK, \]

\[ \text{PUDoF}(B) = \frac{4B - 1}{4B} \]

Recall that \( |T_i| \leq M, \forall i \in [K] \Rightarrow \text{PUDoF}(M) = \frac{2M}{2M+1} \)
Coding Scheme: $B = 1$

$B = \frac{2}{3}$, PUDoF = $\frac{2}{3}$

$\frac{3K}{8}$ users

PUDoF ($B = 1$) = $\frac{3}{4}$

$B = \frac{6}{5}$, PUDoF = $\frac{4}{5}$

$\frac{5K}{8}$ users
Application in Denser Networks

$Tx_i$ is connected to receivers $\{i, i + 1, \ldots, i + L\}$.

Result: Using only zero-forcing transmit beamforming and fractional reuse:

$$PUDoF(L, B = 1) \geq \frac{1}{2}, \forall L \leq 6.$$ without need for interference alignment and symbol extensions

$L = 2$
Application in Denser Networks

\[ \text{PUDoF}(M = 1) = \frac{1}{2} \quad \text{PUDoF}(B = 1) \geq \frac{5}{9} \]
Interference in Cellular Networks

Locally (partially) connected interference channel!
Interference Graph for Single Tier

Each node represents a Tx-Rx pair
No Intra-sector Interference
Partition into Noninterfering Tx-Rx Pairs
Full Cooperation \((M = 6)\)

\[
B = \frac{6 \times 6}{9} = 4; \quad \text{PUDoF} = \frac{6}{9} = \frac{2}{3}
\]
Under-utilizing Backhaul Resources \( (M = 2) \)

\[
B = \frac{6}{9} = \frac{2}{3}; \quad \text{PUDoF} = \frac{4}{9}
\]
No Extra Backhaul Load

Backhaul Load $B = 1$, PUDoF$ = \frac{7}{15}$
Discussion: Cooperation through Backhaul

- Similar gains in DoF for other cellular interference models, with only zero-forcing and fractional reuse
- Gains improve with asymmetric cooperation and interference aware message assignment
- Gains in DoF can also be obtained for uplink with decoded messages being exchanged through backhaul [Ntranos et al ’14]
  - Requires multiple antennas at both mobiles and basestations
  - For same backhaul load factor, gain is smaller than on downlink with Tx cooperation
Summary

- Infrastructure enhancements in backhaul can be exploited through cooperative transmission to lead to significant rate gains
  - Minimal or no increase in backhaul load
  - Fractional reuse and zero-forcing transmit beam-forming are sufficient to achieve rate gains
  - No need for symbol extensions and interference alignment

- Open Questions:
  - Partial/unknown CSI
  - Network dynamics and robustness to link erasures
  - Joint design with message passing schemes for uplink
References


We are writing a book!